The Thesis Committee for Babafemi Anthony Ogunyomi
Certifies that this is the approved version of the following thesis:

Petroleum Development Optimization under Uncertainty: Integrating Multi-Compartment Tank Models in Mixed Integer Non-Linear Programs

APPROVED BY
SUPERVISING COMMITTEE:

Supervisor: ______________________________
Christopher J. Jablonowski

____
Larry W. Lake

Petroleum Development Optimization under Uncertainty: Integrating Multi-Compartment Tank Models in Mixed Integer Non-Linear Programs

by

Babafemi Anthony Ogunyomi, B.Sc.

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Dedication

I would like to dedicate this work to the

loving memory of my mom, late Mrs. O. A. Ogunyomi nee Erubami. I love you mom and

I sincerely miss you.

Acknowledgements

I will like to express my sincere gratitude to Dr. Christopher Jablonowski for his guidance, support and understanding throughout the duration of this work. I will also like to say a big thank you to Dr. Larry Lake for sharing his expertise with me during the course of completing this work. I am also grateful for the great opportunity that this world class institution, the University of Texas at Austin has given me to develop myself particularly I am greatly indebted to faculty and staff of the Department of Petroleum and Geo-systems Engineering for making my experience in the department a memorable one. Overall it has been an enjoyable and rewarding experience which I will like to experience again.

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Abstract

Petroleum Development Optimization under Uncertainty: Integrating Multi-Compartment Tank Models in Mixed Integer Non-Linear Programs

Babafemi Anthony Ogunyomi, M.S.E.

The University of Texas at Austin, 2010

Supervisor: Christopher J. Jablonowski

A field development plan is an important document used to tell the share holders and investors that every aspect of the project has been carefully evaluated. The field development plan should include the objectives of the development, petroleum engineering data, operating and maintenance principles, description of engineering facilities, cost and manpower estimates, project planning and budget proposal. But to arrive at decisions concerning the contents of the field development plan many concept and ideas would have to be screened so that the best ideas and concepts are carried forward for detailed analysis. This screening process can be daunting as there is no limit to the number of viable concepts and ideas. To add to this, for a new field there is hardly ever enough data to fully characterize the reservoir at the time the field development plan is being formulated because there are only a handful of wells in the reservoir. This lack of information about the reservoir introduces uncertainty in the

analysis done during the screening process of the concept selection and can have a significant impact on the quality of the project.

In this work, we present a simple integrated asset model that can be used in conjunction with a proposed framework at the concept screening and selection phase of a project to evaluate the impact of uncertainty in the input variables on key project drivers. The model can be used to screen multiple concepts to arrive at a few promising concepts that point the direction for detailed studies. The application of the model is demonstrated with synthetic cases formulated for a deep water field which is at the concept selection phase. In the demonstration, we investigated how uncertainty in the reservoir thickness (NTG) and the degree of heterogeneity affect the optimal choices for initial facility size, the number of rigs and the number of pre drilled wells.

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CHAPTER ONE: INTRODUCTION

Overview

Field development planning can simply be thought of as the process of making decisions on components that are essential to the operation of a field i.e. ordering and scheduling of investment and operations. It typically involves deciding on the size of the facility to install, gas and water handing capacity, what type of facility to install, number of wells to drill, where and when to drill the wells and a host of other pertinent decisions. Given the huge capital involved the economic success and/or failure of a project greatly depend on the quality of decisions made during the field development phase of the project. Quality as used here means that good decisions are made at the right time.

The Problem of Petroleum Development Optimization under Uncertainty

A field development plan is a key document used to achieve proper communication, discussion and agreement on the activities required for the development of a new field or extension to an existing development (Jahn, 2003). The main purpose of a field development plan is to show a conceptual project specification for subsurface and surface facilities and the operational and maintenance philosophy required to support a proposal for the required investments. It should prove to management and shareholders that every aspect of the project has been thought of and that project drivers have been identified and discussed among the relevant technical and non technical groups. The field development plan should include the objectives of the development, petroleum engineering data, operating and maintenance principles, description of engineering facilities, cost and manpower estimates, project planning and budget proposal. After an oil reservoir is discovered and appraised the next step would be to develop the field. Field development planning essentially involves making decisions concerning the design and operations of the field. These decisions are typically centered around the type of production facility to install, number of platforms, platform size, number of wells (production and injection), location of platforms and wells, number of subsea well heads, production and injection rate over time, improved oil recovery (what type, when, how) etc. Considering the huge capital involved in implementing the field development plan, the success of a project depends on how good the decisions made during the field development stage are (Vasantharajan, 2006). Hence, it is important that these decisions are as good as possible.

In a new oil field development there is hardly ever enough information to develop a concise and thorough field development plan because there are only a few wells in the reservoir at the time the field development plan is being written and there are no production data from which a detailed description of the reservoir can be made. Even in some mature fields the data set available is not enough to completely describe the reservoir. This scarcity of information introduces uncertainty into the concept selection phase of a project which is propagated through to the field development plan (Volz, 2008). A likely consequence of this lack of information is the construction of a facility that is sub-optimal in some attribute, thereby affecting the maximum NPV that can be derived from the field.

Research Objectives

The objective of this study is to develop an integrated asset model to be used as a screening tool for uncertainty analysis for oil field development planning and optimization. To achieve this goal a simple multi-compartment tank reservoir model is coupled to a facility and economic model in an optimization model. This integrated model is then used within an uncertainty analysis workflow to evaluate the impact of uncertainty in input variables on the field development plan. The result produced from the integrated model can provide an insight as to where a more detailed study can begin.

Method

In order to achieve the research objectives, the following systematic approach is utilized:

1. Give a basic overview of field development planning and highlight the main sources of uncertainty
2. Define and explain the fundamental concepts of the reservoir tank model and also provide an overview of mathematical optimization theory
3. Develop the integrated asset model by formulating a maximization problem using net present value (NPV) as the objective function and using a coupling of the reservoir tank model, the surface facility and economic model as the constraints
4. Propose a workflow for uncertainty analysis
5. Demonstrate the integrated model and workflow in an uncertainty analysis for a hypothetical oil field development. The analysis will focus on initial facility size, optimum pre-drilled wells and rig count.

Literature Review

The ordering and scheduling of investment and operations have a significant impact on the financial performance of a field (Vasantharaja, 2006). Planning is complicated because the analysis is affected by the quantity and quality of information which, typically, is not perfect and thereby introduces uncertainty in the field development plan. Several authors have studied the benefits of using optimization theory and integrated asset models in the formulation of field development plans and a survey of the literature revealed that these authors can be separated in to two main categories. The two main categories are:

1. Hypothesis category: This group proposed the application of integrated asset models in an optimization framework to assist in decision making.
2. Application category: This group showed that the application of an integrated asset model in an optimization framework can actually help in decision making.

The application category can further be split into three groups, the first group focused on gas storage projects, the second group focused on gas production fields while the third group focused on oil production fields. The following paragraphs provide a brief summary of these works.

McFarland, Lasdon, and Loose (1983) showed that it is possible to apply generalized reduced gradient nonlinear programming in planning reservoir production policies. They used a tank model in an optimization model to investigate the optimal drilling and production strategy. The model used after tax revenue as the objective function. Their study demonstrated the ability of the model with a simple gas reservoir supported by a water drive and later applied it to an oil and gas reservoir. Their model setup was to maximize the discounted after tax profit subject to reservoir, drilling and deliverability constraints. To solve the resulting problem they reduced it to a three state optimal control problem by converting the drilling rate constraint to a piecewise constant function and solved it using a generalized reduced gradient code. For the two examples used in the study, the results generally agreed with the expectations given the input data.

Lasdon (1984) and McFarland, Lasdon, and Loose (1983) presented the application of reservoir simulation models and numerical optimization techniques to define optimal operating policies for hydrocarbon reservoirs. Their study was focused on gas storage reservoirs. The objective function, defined as either the total deliverability from the field at a specified time or to meet a specified demand schedule was maximized subject to demand, well deliverability and well flow non-negativity constraints. The solution approach utilized the reservoir simulator to obtain pressures and potentials and a reduced gradient algorithm solves for the lagrangian multipliers. For this study the lagrangian multipliers obtained at an optimal solution provide a guide to where new wells should be drilled in the field. The study also used an interior penalty option to guarantee that the deliverability constraints are strictly satisfied at each stage of the optimization process.

Vasantharajan, Al-Hussainy, and Heinemann (2006) applied an integrated model and non linear programming to the development of a gas complex. The model coupled surface, subsurface and economic models together. The study demonstrated the importance of ordering and scheduling of investment and operations on the financial performance of an asset and also showed that the integration of the entire physical system and its financial performance are important when optimizing asset value.

Lund (2000) described the application of a stochastic dynamic programming model and used it to show the importance of flexibility in offshore petroleum projects. The model used a tank type representation of the oil reservoir. His presentation assessed both market (oil price) and reservoir uncertainty.

Aronofsky and Williams (1963) applied linear programming techniques to solve the problem of oil production scheduling from different reservoirs with single and multi-well systems with a finite handling facility capacity. They also formulated a linear programming problem for scheduling drilling and rig operations.

Mantini and Beyer (1979) used optimal control theory to define several physical and economic optimal production and injection rates in a gas reservoir. Their study used a simple tank representation of the reservoir, the deliverability and injection rate equations and the ideal gas law.

Begg, Bratvold, and Campbell (2001) proposed a concept of stochastic integrated asset model (SIAM) in which simplified models are used to model the physical systems present in an asset while the dependence between the physical systems are rigorously represented. The SIAM concept incorporates system models with Monte Carlo simulation embedded with decision analysis tools. The idea is to have better assessment of the impact of uncertainty on the investment and decision making. It aims to take as broad a view as possible of the asset by focusing on the impacts of uncertainty and then doing a detailed analysis when it is justified that it does have a significant impact on decision making.

Bilderbeck and Beck (2005) described the development and application of a spreadsheet based integrated field development planning tool. The tool has a reservoir model that accounts for the uncertainty in reservoir performance and a field development planning model that handles the scheduling and costing of wells and processing facilities. Their study investigated the advantages of multi-stage phasing of the facility construction against single phase up front construction of the facility and also investigated the impact of uncertainty in the unit cost of wells and facility arising from fluctuations in market demand for oilfield services. The tool was applied to an oilfield development in the Middle East.

Haugland, Hallefjord, and Asheim (1988) also described the application of linear programming and a simplified representation of the reservoir to answer the question of production scheduling from an oil reservoir and also showed that the approach could be extended to make decisions concerning field develop. Such decisions include platform capacity and the drilling program (including when and where wells should be drilled). The extended study used mixed integer programming.

Hansen, et el (1992) presented the problem of the location and sizing of offshore platforms for oil exploration as a multi-capacitated plant location problem. The problem was formulated as follows “given a set of oil wells to be drilled and a set of possible locations for platforms of standard sizes, determine the location and capacity of the platforms to be built as well as the assignment of wells to platforms from which they will be drilled, in order to minimize investment costs”. They presented the solution to this problem by mixed integer programming and by a tabu search heuristic and compared the results obtained from each method.

There are numerous other applications of mathematical programming in petroleum engineering for planning operational and development activities such as identifying optimal well location, optimal injection rates in a water-flood etc. The work presented is this thesis is different from the previous works in the following regard:

a. Uncertainty analysis is introduced through the use of probability distribution functions to account for uncertainty in subsurface variables that are not exactly known at the concept selection and screening phase of a project.
b. Heterogeneity within the reservoir is being modeled with a multi-compartment tank representation of the reservoir i.e. the reservoir is divided into different compartments, with each compartment representative of the quality of that portion of the reservoir.

Thesis Description

Chapter one presented a brief background to the problem of field development planning under uncertainty and also stated the goals of this study. In chapter two a case is made for the application of simplified reservoir models at the concept selection phase of oil and gas development projects and the derivation of these reservoir tank models is also presented. A succinct introduction to mathematical programming techniques for linear and non-linear programming as well as integer and mixed integer non-linear programming is also presented. Chapter three indentifies the main components of the integrated optimization model and explains how they were coupled together. In chapter four the results from synthetic cases that were used to test the model are presented. And in chapter five the proposed framework used to evaluate uncertainty is presented. The framework is used in conjunction with the integrated optimization model to assess the impact of uncertainty on decisions concerning the initial facility size, optimum predrilled well and a rig count. Chapter six presents the conclusion and proposes the direction for future work.

CHAPTER TWO: RESERVOIR TANK MODELS AND MATHEMATICAL PROGRAMMING

Overview

A model is a collection of mathematical expressions that describes the essential physical processes that take place in a system. In a petroleum reservoir the typical processes that occur are fluid flow and mass transfer between fluid phases. Hence a model to describe an oil reservoir should account for these processes. Fluid flow in a porous medium is modeled using Darcy's law and mass transfer is modeled using Fick's law. A reservoir model couples all these models together to form a complete description of the reservoir.

A tank, or zero dimensional, model is a fundamental representation of the reservoir that assumes the reservoir is a single homogenous compartment that has properties that are equivalent to the average properties of the actual reservoir, i.e. it neglects the spatial variation in reservoir properties. In certain decision settings this approach may be adequate to solve the problem when the reservoir is almost homogeneous and/or detailed information about the reservoir is not available e.g. during exploratory/appraisal phase of a project. This model is simple and fast-solving and facilitates scenario analysis. It also facilitates the integration of reservoir phenomena in conventional optimization models which is the ultimate objective of this research. More complex reservoir models can be thought of as being composed of multiple compartment tank models.

Reservoir Tank Models

1 Producer–1 Tank (1P–1T Case)

Tank models are based on the principle of conservation of mass, which states that the net increase in or accumulation of mass in a system (or control volume) is equal to the difference between the mass entering and leaving the system (Walsh, 2003). Applying this principle to an element of the reservoir in one dimension results in the continuity equation in one dimension as follows.

$$\frac{\partial(\rho\varphi)}{\partial t} = -\frac{\partial(\rho u)}{\partial x} \tag{2.1.1}$$

Where ρ is the fluid density, φ is the reservoir porosity, u is the interstitial fluid velocity, x is the distance along the x-direction and t is time.

Multiplying Eq. 2.1.1 by the volume of the differential element, $Lhdx$ and noting that $\rho = \frac{\rho_s}{B}$, where $\rho_s$is mass density at standard conditions and B is the formation volume factor (FVF) and integrating would produce the macroscopic mass balance (Walsh, 2003).

$$v_b \frac{d}{dt}\left(\frac{\overline{\varphi}}{B}\right) + q = 0$$

v_b: Reservoir bulk volume, rb

$\frac{\overline{\varphi}}{B}$: Volume weighted average of the ratio of porosity and formation volume factor

Note: $\frac{\overline{\varphi}}{B} = \frac{\int_{x=0}^{x=X} Lh\left(\frac{\varphi}{B}\right)dx}{\int_{x=0}^{x=X} Lhdx} = \frac{\int_{x=0}^{x=X} Lh\left(\frac{\varphi}{B}\right)dx}{V_b}$

$$v_b \frac{\bar{\emptyset}}{B} = \int_{x=0}^{x=X} Lh \left(\frac{\varphi}{B}\right) dx$$

$$q = \int_{x=0}^{x=X} Lh \; d\left(\frac{u}{B}\right), \text{since } q = Lhu$$

The macroscopic mass balance can also be derived using direct statements (Dake, 1978; Ahmed, 2005).

Specifying the macroscopic mass balance equation for each of the different components in the reservoir and performing some mathematical manipulations produces the pressure equation as follows (Walsh, 2003).

$$v_p c_t \frac{dp}{dt} = -q \qquad (2.1.2)$$

Where,

v_p: Reservoir pore volume, bbl

$$c_t = c_o S_o + c_w S_w + c_g S_g \qquad (2.1.3)$$

c_t: Saturation weighted total compressibility, psi^{-1}

q: Net production rate, bbl/d

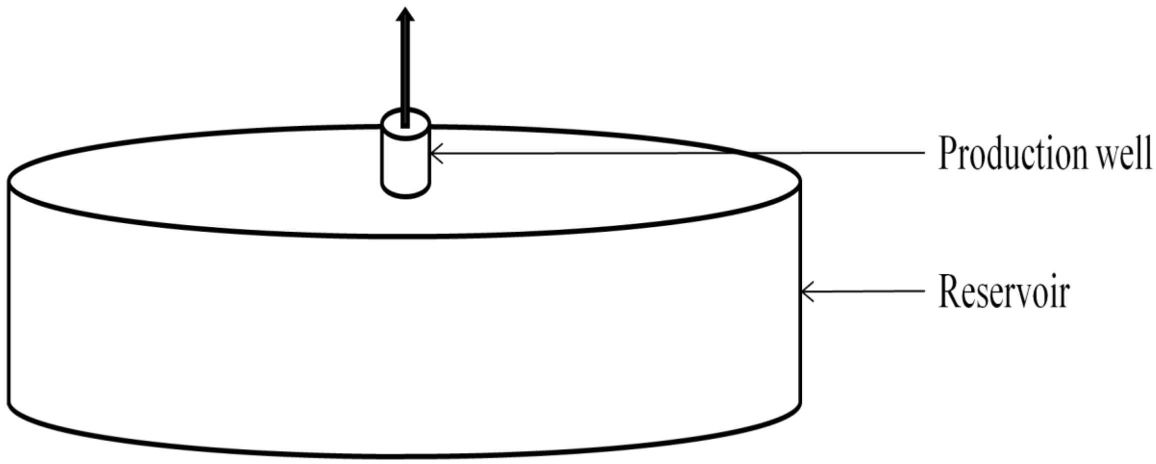

Figure 1: Simplified representation of a reservoir as a tank with one compartment and a production well

For a reservoir that contains water and oil and producing only oil with a solution gas drive, the pressure equation can be integrated subject to the following assumptions.

- The reservoir is producing above the bubble point so that the total compressibility is defined by $c_t = c_o S_o + c_w S_w$. c_t can be regarded as a constant because oil and water are slightly compressible fluids *i.e.* $c_t \Delta p \ll 0.1$ (Dake, 1978).
- Net production is constant within a given time interval.
- The water phase is immobile.

Upon integration the following equation is obtained.

$$P^t = P^0 - \frac{{N_p}^t}{(v_p c_t)} \tag{2.1.4}$$

Where,

P^t:Average reservoir pressure at any time t, psi

P^0:Average reservoir pressure at time t = 0 (initial reservoir pressure), psi

${N_p}^t$:Cumulative oil production at anytime t, stb

The production rate from individual wells in the reservoir is obtained by using the deliverability equation, shown below. The equation assumes that a pseudo steady state flow condition exists in the reservoir.

$$ {q_o}^t = \frac{j(P^t - {P_{wf}}^t)}{B_o} \tag{2.1.5} $$

$$ j = \frac{0.00708kh}{\mu_o\left(\frac{1}{2}\ln\left(\frac{A}{r_w^2 C_A}\right) + 5.75 + s\right)} \tag{2.1.6} $$

Where,

j: Productivity index for well, rb/day/psi

${P_{wf}}^t$: Well flowing bottomhole pressure at time t, psi

${q_o}^t$:Oil production rate at time t, stb/d

μ_o: Oil viscosity, cp

A: Drainage area of the well, acres

C_A: Dietz shape factor

r_w: Well radius, ft

s: Skin factor

The cumulative oil production is obtained from the deliverability equation by using the size of the time step as shown below

$$N_p{}^t = \sum_{time} \sum_{l=1}^{n_w} (q_o{}^t \Delta t) \tag{2.1.7}$$

Where,

l:Index to capture the total number of wells

1 Producer–1 Injector–1 Tank (1P–1I–1T Case)

Depletion drive reservoirs typically recover only a small fraction of the in-place hydrocarbons (10 – 12%). To increase the recovery factor, operators can use water flooding for pressure maintenance. A tank model of this process requires the addition of a source term to the continuity equation presented in the previous section. Equation 2.2.1 is the pressure equation for this process.

$$v_p c_t \frac{dp}{dt} = -q_{producer} + q_{injector} \tag{2.2.1}$$

Here:

$q_{producer}$: Total production rate, stb/d

$q_{injector}$: Total injection rate, stb/d

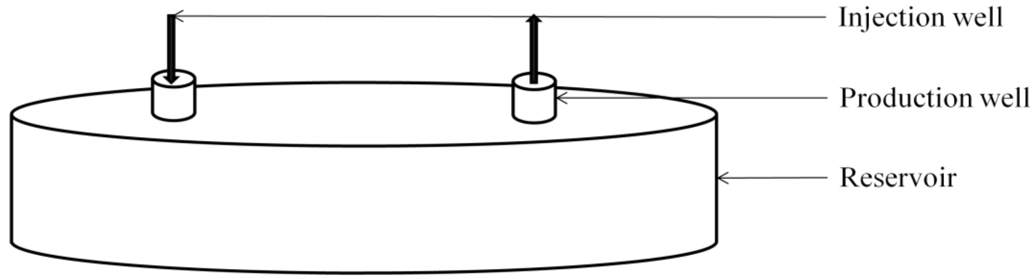

Figure 2: A simplified representation of a reservoir as a tank with one compartment with two well an injection and a production well

The inclusion of an injection well changes the flow dynamics of the system because two phase flow exists in the reservoir. With water injection, saturation changes in the reservoir over time. The water saturation gradually increases and the oil saturation decreases. A consequence of saturation change is a reduction in the ease with which oil flows from the reservoir. This effect is normally captured by introducing the concept of relative permeability in the flow equations. For this study the Corey relative permeability functions (Goda, 2004) are used to describe the two phase flow of water and oil. A tank model representation of the reservoir in this situation is summarized below.

$$P^t = P^0 - \frac{\left({N_p}^t + {W_p}^t - {I_w}^t\right)}{\left(v_p c_t\right)} \tag{2.2.2}$$

$${q_o}^t = \frac{j_o\left(P^t - {P_{wf}}^t\right)}{B_o} \tag{2.2.3}$$

$${q_w}^t = \frac{j_w(P^t - {P_{wf}}^t)}{B_w} \tag{2.2.4}$$

$$i_w{}^t = \frac{j_w{}^o(P^t{}_{inj} - p^t)}{B_w}$$

$q_w{}^t$: water production rate at time t, stb/d

$i_w{}^t$: water injection rate at time t, stb/d

$P^t{}_{inj}$: injection pressure, psi

The productivity index is modified by introducing the relative permeability, k_{ri}.

$$j_i = \frac{0.00708hkk_{ri}}{\mu_i\left(\frac{1}{2}\ln\frac{A}{r_w^2C_A} + 5.75 + s\right)} \tag{2.2.5}$$

i = oil or water. $j_w{}^o$ is the injection well injectivity index and it is computed with equation 2.2.5 with the relative permeability equal to the end point water relative permeability. Cumulative production and injection is computed as follows.

$$N_p{}^t = \sum_{time}\sum_{l=1}^{nw}(q_o{}^t\Delta t) \tag{2.2.6}$$

$$W_p{}^t = \sum_{time}\sum_{l=1}^{nw}(q_w{}^t\Delta t) \tag{2.2.7}$$

$$I_w{}^t = \sum_{time}\sum_{l=1}^{iw}(i_w{}^t\Delta t)$$

$W_p{}^t$: Cumulative water production at anytime t, stb

$I_w{}^t$: Cumulative water injection at anytime t, stb

The relative permeability functions for two phase flow of water and oil are defined as follows.

$$k_{ro} = k_{o,endpoint}(1 - S)^n \tag{2.2.8}$$

$$k_{rw} = k_{w,endpoint}S^m \tag{2.2.9}$$

Where,

$k_{o,endpoint}$:Endpoint relative permeability to oil

$k_{w,endpoint}$:Endpoint relative permeability to water

S:Normalized water saturation

$$S = \frac{S_w - S_{wr}}{1 - S_{wr} - S_{or}} \tag{2.2.10}$$

S_w: Average water saturation, dimensionless

S_{wr}: Residual water saturation, dimensionless

S_{or}: Residual oil saturation, dimensionless

The water saturation is updated at the end of every time interval with the following equation.

$$S_w = S_{oi} - \frac{N_p}{v_p} \tag{2.2.11}$$

Where,

S_{oi}: Initial oil saturation, dimensionless

N Producers–M Injectors–P Tanks (NP–MI–PT Case)

This section generalizes the model for *N* number of producers, *M* injectors and *P* tanks. The discrete form of the pressure equation can be written as

$$v_p c_t \left[\frac{p_i^{n+1} - p_i^n}{\Delta t}\right] = -q \text{ for } i = 1,2,\dots,P \tag{2.3.1}$$

The subscript i and superscript n denotes the compartment and time levels respectively. The cross-flow terms $q_{xf_{ij}}$ are obtained from

$$q_{xf_{ij}} = -T_{ij}\left[p_i - p_j\right] \tag{2.3.2}$$

$$T_{ij} = \frac{2T_iT_j}{T_i + T_j} \tag{2.3.3}$$

$$T_i = \frac{0.00708 k_i A_i}{\mu L_i}$$

Where,

T_{ij}: Transmissibility between compartment i and j, bbl/d/psi

T_i: Transmissibility for compartment i, bbl/d/psi

L_i: Length of compartment i, ft

A_i: Cross sectional area of compartment i, ft^2

Transmissibility should be computed for the different components flowing between compartment i and j because k_i is the product of the permeability of compartment i and the relative permeability of the cross flowing component.

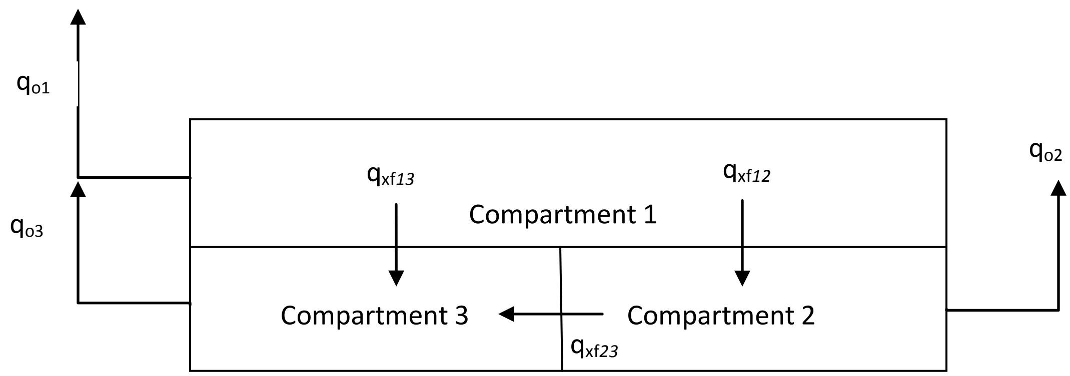

Figure 3: A simplified representation of a reservoir as a tank with three compartments

Writing the discrete form of the pressure equation for each compartment produces a system of equations that can be solved implicitly or explicitly for pressure. For figure 3, the system of equations in the implicit case using vector notation is as follows.

$$-\overline{\overline{\mathrm{T}}}\vec{\mathrm{p}}^{n+1} = \mathrm{C}\vec{\mathrm{p}}^{n} - \mathrm{j}\mathrm{p}_{wf} \tag{2.3.4}$$

Where,

$$\overline{\overline{\mathrm{T}}} = \begin{bmatrix} \left(j_1 + T_{12} + T_{13} - \frac{v_{p1}c_t}{\Delta t}\right) & -T_{12} & -T_{13} \\ -T_{12} & \left(j_2 + T_{12} + T_{23} - \frac{v_{p2}c_t}{\Delta t}\right) & -T_{23} \\ -T_{13} & -T_{23} & \left(j_3 + T_{13} + T_{23} - \frac{v_{p3}c_t}{\Delta t}\right) \end{bmatrix}$$

$$\vec{\mathrm{p}}^{n+1} = \begin{bmatrix} p_1^{n+1} \\ p_2^{n+1} \\ p_3^{n+1} \end{bmatrix}$$

$$C = \begin{bmatrix} \frac{v_{p1} c_t}{\Delta t} & 0 & 0 \\ 0 & \frac{v_{p2} c_t}{\Delta t} & 0 \\ 0 & 0 & \frac{v_{p3} c_t}{\Delta t} \end{bmatrix}$$

$$\vec{p}^n = \begin{bmatrix} p_1^n \\ p_2^n \\ p_3^n \end{bmatrix}$$

and

$$jp_{wf} = \begin{bmatrix} j_1 p_{wf1} \\ j_2 p_{wf2} \\ j_3 p_{wf3} \end{bmatrix}$$

Note that v_{pi} represents the pore volume of compartment i.

For the explicit case, the discrete form of the pressure equation for each compartment in vector notation is shown in equation 2.3.5.

$$C\vec{p}^{n+1} = \overline{\overline{T}}\vec{p}^n - jp_{wf} \qquad (2.3.5)$$

Where $\overline{\overline{T}}$ in this case is

$$\overline{\overline{T}} = \begin{bmatrix} \left(j_1 + T_{12} + T_{13} + \frac{v_{p1} c_t}{\Delta t}\right) & -T_{12} & -T_{13} \\ -T_{12} & \left(j_2 + T_{12} + T_{23} + \frac{v_{p2} c_t}{\Delta t}\right) & -T_{23} \\ -T_{13} & -T_{23} & \left(j_3 + T_{13} + T_{23} + \frac{v_{p3} c_t}{\Delta t}\right) \end{bmatrix}$$

The size of the matrix depends on the number of compartments present.

A second approach to the multi-tank formulation is to simply write equation 2.2.2 for each compartment and adding a cross-flow term in each equation. The cross-flow term should be added to each equation to account for cross-flow between adjoining tanks.

$$P^t{}_i = P^0{}_i - \frac{\left(N_p{}^t{}_i + W_p{}^t{}_i - I_w{}^t{}_i + N_{xf}{}^t{}_{ij}\right)}{\left(v_{p_i} c_{t_i}\right)}$$

Where $N_{xf}{}^t{}_{ij}$ is the cumulative cross-flow between adjacent tanks within the time interval Δt and the subscripts i and j represents the compartment number (subscript ij thus represent the different combination of adjacent tanks).

$$N_{xf}{}^t{}_{ij} = \sum_{time} q_{xf_{ij}} \Delta t$$

All other symbols have their usual meaning. For figure 3 the relevant equations with this approach are shown below.

$$P^t{}_1 = P^0{}_1 - \frac{\left(N_p{}^t{}_1 + W_p{}^t{}_1 - I_w{}^t{}_1 + N_{xf}{}^t{}_{12} + N_{xf}{}^t{}_{13}\right)}{\left(v_{p_1} c_{t_1}\right)}$$

$$P^t{}_2 = P^0{}_2 - \frac{\left(N_p{}^t{}_2 + W_p{}^t{}_2 - I_w{}^t{}_2 - N_{xf}{}^t{}_{12} + N_{xf}{}^t{}_{23}\right)}{\left(v_{p_2} c_{t_2}\right)}$$

$$P^t{}_3 = P^0{}_3 - \frac{\left(N_p{}^t{}_3 + W_p{}^t{}_3 - I_w{}^t{}_3 - N_{xf}{}^t{}_{13} - N_{xf}{}^t{}_{23}\right)}{\left(v_{p_3} c_{t_3}\right)}$$

The resulting system of equations can be solved simultaneously.

Mathematical Programming

Optimization is a vast area of applied mathematics that involves finding the best value of a target variable subject to constraints. In the simplest case, the objective is to find the minimum or maximum of an objective function by systematically and intelligently choosing the values of independent variables from the feasible domain.

$$\begin{gathered} max\ f(x) \\ subject\ to\ g(\boldsymbol{x}), \dots, h(\boldsymbol{x}) \end{gathered} \tag{3.0.1}$$

f(x) is the objective function and *g(**x**),…,h(**x**)* is a system of constraint equations. ***x*** is a vector of independent variables that combine in different quantities to determine the value of the objective function. For example, in a resource allocation problem, the independent variables represent finite resources that must be combined in a given proportion to give the best possible value of the objective function.

There are different subfields of mathematical programming: linear programming, non-linear programming, integer programming, stochastic programming, mixed integer non linear programming, etc. The different names come from the nature of the problem, the functional form of the objective function and/or the constraints.

Linear Programming

Generally in a linear programming problem we seek to determine the optimal value of a linear objective function subject to linear constraints whose form may be equality or inequality. In mathematical symbols:

$$\begin{aligned} &Minimize \quad z'\boldsymbol{x} \\ &subject\ to\ A\boldsymbol{x} = b \\ &\qquad\qquad\quad \boldsymbol{x} \geq 0 \end{aligned}$$

The vector ***x*** is a vector of decision variables. Any vector ***x*** satisfying all the constraints (A***x*** = b) is called a feasible vector (solution) and the set of all feasible solutions is called the feasible set or region. If there is no sign restriction on the decision variables then it is a free variable. The function that we seek to minimize is called the objective function. A feasible vector that minimizes the objective function is called the optimal feasible solution or optimal solution. If for every real number R, there exist a feasible solution ***x'*** such that z'***x'*** < R, then the optimal value is $-\infty$ and the objective function is said to be unbounded below (Lasdon, 1970).

There are different methods of solving a liner programming problem such as graphically, the simplex algorithm, Lagrangian multiplier method, and others. Below is a simple example that is solved using the simplex algorithm and a graphical approach.

$$\text{maximize} \quad x_1 + x_2$$

$$\begin{aligned} \text{subject to} \quad x_1 + 2x_2 &\leq 4 \\ 4x_1 + 2x_2 &\leq 12 \\ -x_1 + x_2 &\leq 1 \end{aligned}$$

$$x_i \geq 0, i = 1,2$$

The shaded region in the graph is called the feasible set and it is defined by the constraints of the problem. We seek a point **x** (x_1, x_2) such that the objective function, $x_1 + x_2 = c$, is a maximum. Looking at the graph, we are interested in a line with a slope of -1 which is as far as possible from the origin and still within the feasible set. This point is identified to be **C** (8/3, 2/3) and the value of the objective function is equal 10/3.

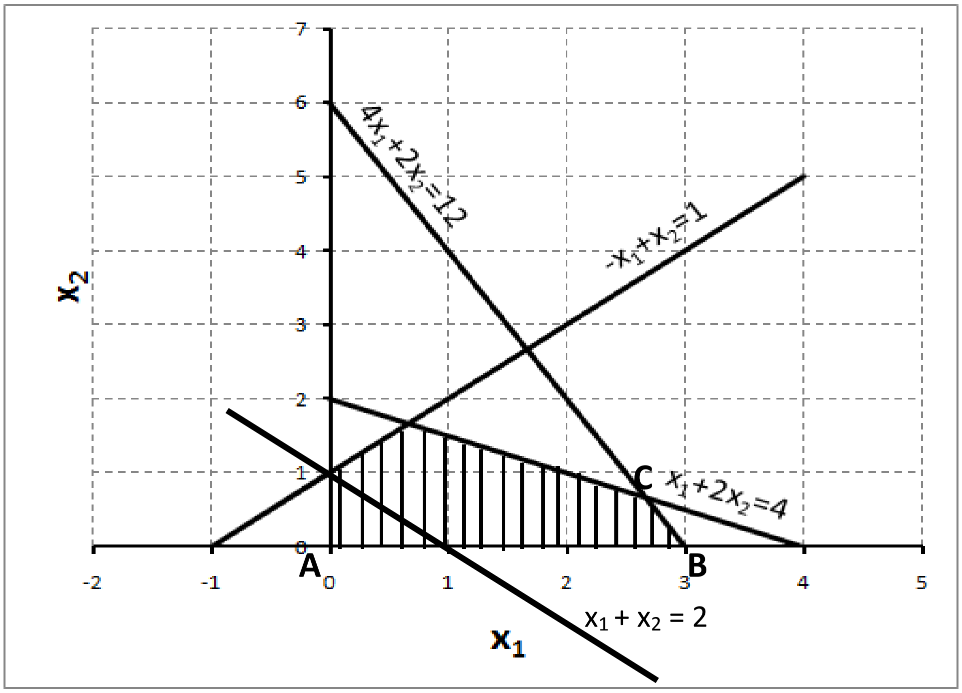

Figure 4: Graphical solution to linear programming example

Next, a tableau implementation of the simplex algorithm is presented using the previous example. Before the simplex algorithm is presented some important definitions are presented. Consider the constraints $\mathbf{Ax} = \mathbf{b}$ and $\mathbf{x} \geq \mathbf{0}$ and assume that the $\mathrm{m} \times \mathrm{n}$ matrix $\mathbf{A}$ has linearly independent rows where $\mathrm{m} \leq \mathrm{n}$, then:

a) A basis matrix $\mathbf{B}$ for matrix $\mathbf{A}$ is a collection of m linearly independent columns $\mathbf{A}_{\mathrm{B(l)}} \ldots \mathbf{A}_{\mathrm{B(m)}}$ of matrix $\mathbf{A}$. Because the basis matrix is a full rank matrix it can be inverted.

b) A basic solution $\mathbf{x_B}$ of $\mathbf{Ax} = \mathbf{b}$ is obtained from $\mathbf{x_B} = \mathbf{B^{-1}b}$. Where the remaining $(\mathrm{n}-\mathrm{m})\ \mathrm{x}$ variables are set equal to zero. $\mathbf{x_B}$ is called the basic variables and the remaining $(\mathrm{n}-\mathrm{m})\ \mathrm{x}$ are called non basic variables.

The structure of the simplex algorithm tableau is presented below in Table 1.

Table 1: Structure of the simplex algorithm tableau

$-\boldsymbol{z}_B{}'\mathbf{B}^{-1}\mathbf{b}$	$\boldsymbol{z}' - \boldsymbol{z}_B{}'\mathbf{B}^{-1}\mathbf{A}$
$\mathbf{B}^{-1}\mathbf{b}$	$\mathbf{B}^{-1}\mathbf{A}$

In Table 1, the column $\mathbf{B}^{-1}\mathbf{b}$ (the entry at the bottom left corner of Table 1) is called the zeroth column and it contains the values of the basic variables, $\mathbf{x_B}$. The first row of Table 1 is called the zeroth row and the entry at the top left corner contains the value $-\mathbf{z_B}'\mathbf{x_B}$ ($\mathbf{z_B}$ is a vector of the coefficient of the basic variables in the objective function) which is the negative of the current objective value. The remaining entry of the zeroth row (i.e. the entry at the top right corner of Table 1) is the row vector of the reduced cost. The reduced cost is defined as $\boldsymbol{z}' - \boldsymbol{z}_B{}'\mathbf{B}^{-1}\mathbf{A}$ ($\boldsymbol{z}$ is the vector of the coefficients of the decision variables in the objective function). The column $\mathbf{B}^{-1}\mathbf{A}$ (the entry at the bottom right corner of Table 1) is called the *i*th column of the tableau. The column $\mathbf{u} = \mathbf{B}^{-1}\mathbf{A_j}$ ($\mathbf{A_j}$ is the jth column of matrix A) is the column that corresponds to the variable that enters the basis; it is called the pivot column.

The information within the tableau can be interpreted as follows: the equality constraints are initially given in the form $\mathbf{b} = \mathbf{Ax}$**.** Given the current basis matrix $\mathbf{B}$, these equality constraints can also be expressed in the equivalent form $\mathbf{B}^{-1}\mathbf{b} = \mathbf{B}^{-1}\mathbf{Ax}$. As a result, the rows of the tableau provides the coefficients of the equality constraints $\mathbf{B}^{-1}\mathbf{b} = \mathbf{B}^{-1}\mathbf{Ax}$**.**

The algorithm for the tableau implementation is presented below;

1. A typical iteration starts with the tableau associated with a basis matrix **B** and the corresponding basic feasible solution **x**.

2. Examine the reduced cost ($\bar{\mathbf{z}}_j = \mathbf{z}_j - \mathbf{z}_B{}' \mathbf{B}^{-1}\mathbf{A}_j$) in the zeroth row of the tableau. If they are all nonnegative the current basic feasible solution is optimal and the algorithm terminates; else choose a column $\mathbf{A}_j$ for which the reduced cost is negative.
3. Consider the vector $\mathbf{u} = \mathbf{B}^{-1}\mathbf{A_j}$ which is the jth column (pivot column) of the tableau. If no component of **u** is positive the optimal cost is $-\infty$ and the algorithm terminates.
4. For each row *i* that $\mathbf{u}_i$ is positive, compute the ratio $\frac{x_{B(i)}}{u_i}$ ($x_{B(i)}$ is the basic variable in row *i*). Let *l* be the index of a row that corresponds to the smallest ratio. Then the column $\mathbf{A}_{B(l)}$ exits the basis and the column A_j enters the basis.
5. Add to each row of the tableau a constant multiple of the *l*th row (the pivot row) so that u_l (the pivot element) becomes one and all other entries of the pivot become zero.

Before implementing the algorithm the example problem is converted into standard form by introducing slack variables x_3, x_4 and x_5 as shown below

$$\begin{aligned}
&\text{minimize} \quad -x_1 - x_2 \\
&\text{subject to} \quad x_1 + 2x_2 + x_3 = 4 \\
&\qquad 4x_1 + 2x_2 + x_4 = 12 \\
&\qquad -x_1 + x_2 + x_5 = 1 \\
&\qquad x_i \geq 0, i = 1,2
\end{aligned}$$

Step 1: For this example a basis matrix is the identity matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and the corresponding basic feasible solution is **x** = (0, 0, 4, 12, 1). For this basis, the non basic variables are x_1 and x_2 while x_3, x_4 and x_5 are the basic variables. This is point A on the previous graph and the initial tableau is presented in table 2 below:

Table 2: Initial simplex tableau for example problem

	x_1	x_2	x_3	x_4	x_5
0	-1	-1	0	0	0
$x_3 = 4$	1	2	1	0	0
$x_4 = 12$	4	2	0	1	0
$x_3 = 1$	-1	1	0	0	1

Step 2: Examining the zeroth row (row two) of table 2, the value of the objective function is zero and the reduced cost of x_1 and x_2 is equal to -1 while the reduced cost of $x_{3,}$ x_4 and x_5 is equal to zero. Because the reduced cost of the x_1 and x_2 are negative, either column A_1 (the second column of the tableau) or column A_2 (the third column of the tableau) can be chosen to enter the basis. For this example column A_1 is chosen to enter the basis.

Step 3: The vector $\mathbf{u} = \mathbf{B}^{-1}\mathbf{A}_\mathrm{j}$where j = 1, is $\mathbf{u} = (1, 4, -1)'$**.** Column two of table 2 above is now called the pivot column because it has been chosen to enter the basis. Because two components of the pivot column are positive ($\mathrm{u}_1 = 1, \mathrm{u}_2 = 4$) we proceed to step 4 of the algorithm.

Step 4: For each component of $\mathbf{u}$ that is positive the ratio $\frac{\mathrm{x}_{\mathrm{B(i)}}}{\mathrm{u}_\mathrm{i}}, \mathrm{i} = 1{,}2$ is computed $\frac{\mathrm{x}_{\mathrm{B}(1)}}{\mathrm{u}_1} = \frac{4}{1} = 4$ and $\frac{\mathrm{x}_{\mathrm{B}(2)}}{\mathrm{u}_2} = \frac{12}{4} = 3$. The smallest ratio corresponds to row 4 (i = 2), thus the column corresponding to the second basic variable $\mathrm{x}_{\mathrm{B}(2)} = \mathrm{x}_4$exits the basis matrix i.e. column 5 exits the basis matrix and column two enters the basis matrix. The pivot element is $\mathrm{u}_2 = 4$.

Step 5: A series of row operations are performed until all components above and below the pivot element are equal to zero and the pivot element is equal to one. The resulting tableau is presented in table 3 below.

Table 3: Tableau after first iteration

	x_1	x_2	x_3	x_4	x_5
3	0	-1/2	0	1/4	0
$x_3 = 1$	0	3/2	1	-1/4	0
$x_1 = 3$	1	1/2	0	1/4	0
$x_5 = 4$	0	3/2	0	1/4	1

The basis feasible solution is now **x** = (3, 0, 1, 0, 4) where $x_1 = 3$, $x_2 = 0$, $x_3 = 1$, $x_4 = 0$ and $x_5 = 4$. Note that x_2 and x_4 are now the non basic variables while x_1, x_3 and x_5 are the new basic variables. We have thus moved from point A to point B on the graph.

As was done before, by going through the steps of the algorithm we arrive at location C on the graph. The simplex tableau is presented in table 4 below. Because all the reduced cost are non negative the algorithm terminates and we have an optimal solution to the problem.

Table 4: Optimal simplex tableau for example problem

	x_1	x_2	x_3	x_4	x_5
10/3	0	0	1/3	1/6	0
$x_2 = 2/3$	0	1	2/3	-1/6	0
$x_1 = 8/3$	1	0	-1/3	1/3	0
$x_5 = 3$	0	0	-1	1/2	1

The optimal solution is **x** = (8/3, 2/3, 0, 0, 3) where $x_1 = 8/3$, $x_2 = 2/3$, $x_3 = 0$, $x_4 = 0$ and $x_5 = 3$. This solution is the same as the solution obtained by the graphical method. A detail description of the algorithm can be found in Bertsimas (1997).

Non-linear Programming

As the name implies non-linear programming problems are maximization or minimization problems in which the objective function is non-linear or at least one of the constraint functions is non-linear. In non-linear programming the optimum is not necessarily at an extreme point of the feasible set or constraint set and may not even be at the boundary (Lasdon, 1970). Also the global optimum may be different from the local optimum. Most of the solution algorithms for non-linear programming problems can be grouped into two main groups:

a.) The method of feasible directions

b.) The penalty function methods

The concept of the method of feasible direction is to start at a point **x'** satisfying all constraints and finding a direction **d** such that no constraint is violated when a small step is taken in that direction to a point **x''** in order to improve the objective function. The algorithms in this group terminate when it is not possible to find such a direction **d** that improves the objective function. The direction **d** is called a useable feasible direction. Directions that do not improve the objective function are called feasible directions. There are numerous ways in which the useable feasible or feasible directions can be determined. Examples of this are Zontendijk's procedure and Rosen's gradient projection methods. A disadvantage of these methods is that it is possible for zigzagging[1] to occur.

The penalty function techniques changes a constrained minimization problem to an unconstrained minimization problem by introducing a penalty function. Consider the minimization problem

[1] Zigzagging occurs when during the search for an optimal feasible solution the algorithms goes outside feasible region and has to take a recovery move to get back in to the feasible region. Zigzagging render an optimization algorithm inefficient.

$$\begin{aligned} Minimize \quad & f(\boldsymbol{x}) \\ subject\ to\ A_j\boldsymbol{x} &= b \\ \boldsymbol{x} &\geq 0 \end{aligned}$$

introduce the penalty function

$$z(y) = \begin{cases} 0, & y \geq 0 \\ \infty, & y < 0 \end{cases}$$

Then the original problem can be converted to minimizing L(x) subject to no constraints where L(x) is defined as

$$L(x) = f(x) + \sum z(A_j(\boldsymbol{x}))$$

When all the constraints are satisfied the second term in L(x) is equal to zero and minimizing L(x) is equivalent to minimizing f(x). On the other hand, if any of the constraint is violated the second term penalizes for such violation and minimizing L(x) will not select a point outside of the feasible set. This category of methods also has their problems which are discussed in detailed in (Peressini, 1988; Bazaraa, 2006). Examples of such methods are Fiacco-McCormick method and Duality methods. A simple example is presented below to illustrate a non linear programming problem.

$$\begin{aligned} Minimize \quad & z = (x_1 - 4)^2 + (x_2 - 5)^2 \\ subject\ to \quad & 6 - x_1 - x_2 \geq 0 \\ & -3.5 + x_1 - x_2 \leq 0 \end{aligned}$$

Define penalty functions $\varphi_1$and $\varphi_2$and form the unconstrained minimization problem L(x_1, x_2, φ_1, φ_2), where,

$$L(x_1, x_2, \varphi_1, \varphi_2) = (x_1 - 4)^2 + (x_2 - 5)^2 + \varphi_1(6 - x_1 - x_2) + \varphi_2(-3.5 + x_1 - x_2)$$

Then compute the derivative of L(x_1, x_2, φ_1, φ_2) with respect to x_1 and x_2 and equate to zero.

$$\frac{\partial L}{\partial x_1} = 2(x_1 - 4) - \varphi_1 + \varphi_2 = 0$$

$$\frac{\partial L}{\partial x_2} = 2(x_2 - 5) - \varphi_1 - \varphi_2 = 0$$

Solving for x_1 and x_2

$$x_1 = \frac{(\varphi_1 - \varphi_2)}{2} + 4$$
$$x_2 = \frac{(\varphi_1 + \varphi_2)}{2} + 5$$

Using the constraint equation, the penalty functions are obtained to be $\varphi_1 = -3$ and $\varphi_2 = -4.5$. The corresponding decision variables are x_1 = 4.75 and x_2 = 1.25 and the minimum value of the objective function that satisfies the constraints is $z = 14.625$. The graph of the problem is shown in the figure below

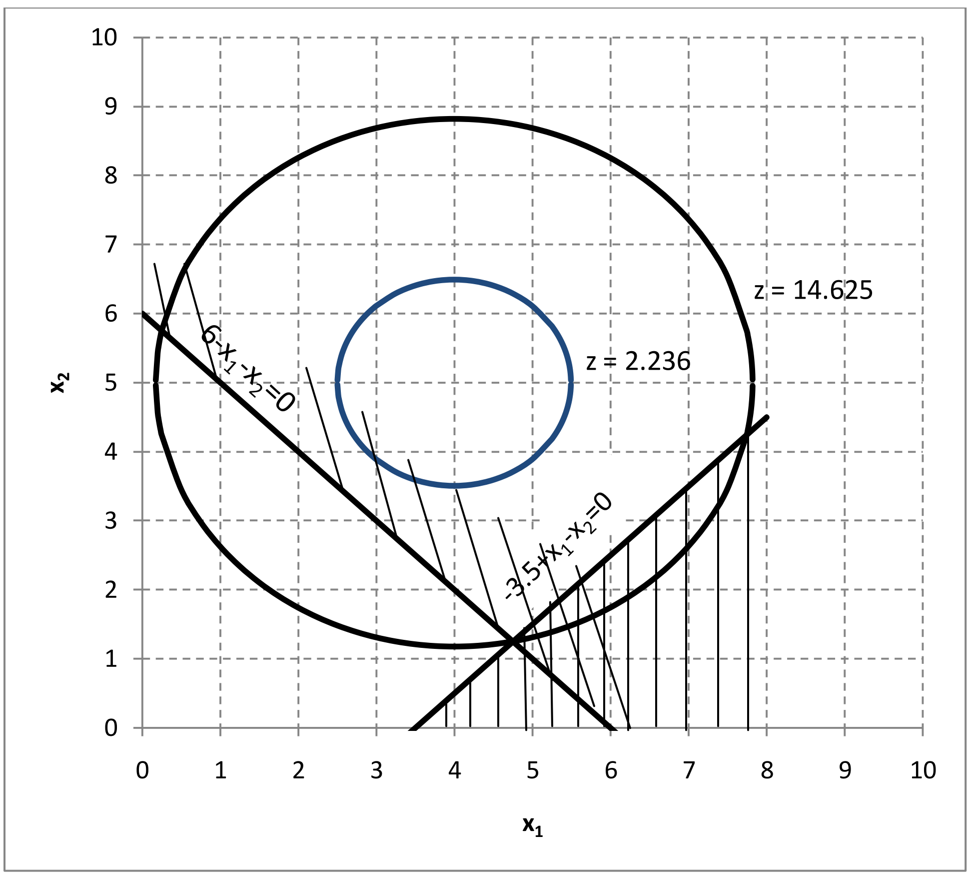

Figure 5: Graphical solution of non-linear programming problem

Integer and Mixed Integer Non-linear Programming

These classes of problems are similar to linear programming problems except that in pure integer programming problems all the variables are integers while in mixed integer programming problems the variables are a mixture of integer and real numbers. These classes of problems are a lot more difficult to solve than linear programming problems. Some well known formulations of integer programming problems are the national science foundation (NSF) scholarship allocation, capital budgeting etc. Examples of integer programming algorithms are the cutting plane methods, branch and bound method, dynamic programming. An exhaustive discussion of these topics can be found in published texts such as (Bertsimas, 1997; Mokhtar, 1979; Mordecai, 2003).

CHAPTER THREE: OPTIMIZATION MODEL

Overview

In this study we have set up the problem as a mixed integer nonlinear program (MINLP) that maximizes the net present value (NPV) of an asset. This can be represented as follows

$$\begin{gathered} max\ f(x,y) \\ subject\ to \\ g(x,y) \leq 0 \\ x \in continuous \\ y \in integer \end{gathered} \tag{3.0.2}$$

Non linearities are introduced into the problem from the relative permeability functions (equations 2.2.8 and 2.2.9) defined to capture two phase flow of oil and water in the reservoir. Integer variables are required to model decision variables that suggest when wells should be drilled and when the expansion option is exercised. The General Algebraic Modeling System (GAMS) was used to solve the optimization problem formulated for this study. GAMS provides fast solutions to large scale problems. In the following section a detailed description of the optimization model is presented.

Model Formulation

The model presented here is an integrated reservoir, economic, and facility optimization model because it couples the reservoir to the facility and uses revenue and cost functions to maximize the NPV. Hereafter, this model is referred to as an integrated asset model (IAM). The model has three main components, namely: 1. Reservoir model 2. Economic model 3. Wells and Facility model. Parameters values specified below define the diagnostic case, and these values are updated in the case study that follows. Conceptually, this can be written as follows.

$$maximize\ NPV = f(N_p, CAPEX, OPEX) \tag{3.1.1}$$

Subject to

$$\begin{gathered} Reservoir\ model \\ Wells\ and\ facility\ model \\ Economic\ model \end{gathered}$$

The reservoir model used in this study is the previously specified tank model. As stated above, the tank model is simple and fast-solving, thus it is ideally suited for integration into optimization models where the purpose is to perform a wide variety of scenario analyses (i.e. numerous iterations).

Economic Model

The optimization model is defined to maximize the objective function which in this study is the net present value. The objective function, *z,* is defined as follows.

$$\begin{gathered} z = \Big[\big[price \times \Delta N_p(t)\big] - TotalOPEX(t) - TotalCAPEX(t)\Big] \\ \times Discountfactor - Platformcost \end{gathered} \tag{3.2.1}$$

Where,

Price: Price of oil, dollars per barrel

$\Delta N_p(t)$: Cumulative oil produced over each time period

TotalOPEX(t): Operating expense, \$/bbl

TotalCAPEX(t): Capital expenditure, \$/bbl

The cumulative oil produced is obtained by multiplying the deliverability equation with the time interval and summing it over the total number of wells producing from the tank.

$$N_p(t) = \sum\nolimits_{prodwell} q_o(t, prodwell) \qquad (3.2.2)$$

prodwell is the index for the production wells. $\Delta N_p(t)$ is the difference between the cumulative oil produced at time *t* and time *t–1* i.e. $N_p(t) - N_p(t-1)$. The operating cost was modeled as follows.

$$TotalOPEX(t) = \; variable\, production\, cost \; \times \Delta \mathrm{N}p(t) \qquad (3.2.3)$$

Variable production cost was assumed constant at 4 USD throughout the field life. TotalCAPEX(t) was defined to include two components, well cost and facility expansion cost.

$$TotalOPEX(t) = \; variable\, production\, cost \; \times \Delta \mathrm{N}p(t) \qquad (3.2.4)$$

Facility cost equations are adapted from a study on deepwater project costs as reported by Jablonowski (2009). Expansion cost is defined as follows.

$$Expansion\ cost(t) = \beta_o \left[[add\_Switch(t) \times \beta_1] + \left[\frac{Xtracap(t)}{1000} \times \beta_2 \right] \right] \; (3.2.5)$$

The coefficients β_0, β_1 and β_2 as used in the base case model are two, 50 million and 4 ½ million respectively. Another version of the equation was used to compute the platform cost.

$$Platform\ cost(t) = \zeta_0 + \left[\zeta_1 \times \frac{FAC_{init}(t)}{1000} \right] + [\zeta_2 \times maxwell\ slot] \qquad (3.2.6)$$

Here the base case coefficients $\zeta_{0,}$ ζ_1 and ζ_2 as used in the base case model are 50 million, 4 ½ million and 22 ½ million respectively. The initial facility size is represented as $FAC_{init}(t)$ and it has units of barrels per day. *Maxwell slot* represents the maximum number of well slots for dry trees on the facility. *Wellcost(t)* was modeled as follows.

$$\begin{aligned} wellcost(t) &= fixedwellcost \\ &\times \left[\sum_{prodwell} droil(t, prodwell) \right. \\ &\left. + \sum_{injewell} drwater(t, injewell) \right] \end{aligned} \tag{3.2.7}$$

fixedwellcost is constant and representative of the average cost of drilling a deep water well, and is set equal to USD 250 million. $\sum_{\text{prodwell}}$ *droil(t,prodwell)* and $\sum_{\text{injewell}}$ *drwater(t,injewell)* is a syntax used in the code to count the total number of production and injection wells drilled at time *t*. *droil(t,prodwell)* and *drwater(t,injewell)* are binary variables defined to indicate when a well is drilled. *prodwell* is the index used to identify production wells, and *injewell* is the index that identifies injection wells.

Facility Model

The coupling of the reservoir to the facility is straightforward and involves common sense constraints. The total production from the reservoir is constrained to be less than or equal to the volumetric capacity of the surface facility.

$$facilitycapacity(t) \geq \sum_{prodwell} q_o(t, prodwell)$$

facilitycapacity(t) is the surface facility capacity at time t in barrels per day and the left hand side is the total volumetric flow of oil from the reservoir. An important aspect of the model development was implementing an endogenous real option to expand the facility size. This is a key feature of the model because it is desired to simulate subsurface properties in a Monte Carlo fashion and we would like the model to account for decision-maker responses to revelations of the uncertain variables. This feature was implemented using a step function as follows.

$$facilitycapacity(t) = \begin{cases} FAC_{init}(t) & t < t_{exp} \\ facilitycapacity(t-1) + extracapacity(t) & t = t_{exp} \\ facilitycapacity(t-1) & t > t_{exp} \end{cases} \quad (3.3.1)$$

The facility capacity is set equal to the initial facility size when $t < t_{exp}$, where t_{exp} represents the time at which facility expansion is allowed. When $t = t_{exp}$, the facility capacity is set equal to the new expanded facility size (it will be larger if expansion option is exercised, or the same as the initial capacity if it is not exercised), and when $t > t_{exp}$ the facility capacity is equal to the expanded facility size. *facilitycapacity(t)* and *extracapacity(t)* are in units of barrels per day.

Wells are modeled using a two dimensional matrix of binary variables. The first dimension is time and the second is the well number. To ensure that no well is drilled twice, the following equation is required.

$$\sum_{t} dr_i(t,n) \leq 1 \tag{3.3.2}$$

Index *i* represents oil or water. The total number of wells drilled in a given period is constrained by the drilling capacity (i.e. number and productivity of drilling rigs) as follows.

$$\sum_{n} dr_i(t,n) \leq no\ of\ wells \tag{3.3.3}$$

The following two equations were defined to ensure that production can occur only from a well that has been drilled.

$$wellcap(t,n) = maximumratedr_i(t,n) \times \sum_{1}^{t} dr_i(t,n) \tag{3.3.4}$$

$$q_i(t,n) \leq wellcap(t,n) \tag{3.3.5}$$

maximumrate is a constant and denotes the maximum production rate (in *bbl/d*) from a well. *wellcap(t,n)* on the other hand is a variable that guarantees that once a well is drilled it has a capacity from which oil can be produced.

In setting up the optimization model the flowing bottomhole pressure was set up as a free variable that has a minimum value equal to the hydrostatic pressure exacted by a column of fluid in the wellbore.

$$P(t) \geq P_{hp} \tag{3.3.6}$$

P_{hp} is the hydrostatic pressure exacted by a column of fluid in the wellbore, for this study it is approximately equal to 9000 *psi*.

The reservoir pressure is also constrained to be less than the initial reservoir pressure to avoid fracturing the formation.

$$P(t) \geq P_i \tag{3.3.7}$$

Reservoir Model

A tank type representation of the reservoir was used in the IAM and this has been discussed in chapter two.

Decision Variables

The main decision variables are the number of rigs, the number of pre-drilled wells and the initial facility size. These variables are of most interest because of their large impact on the capital cost. Also, the decisions are made prior to the resolution of the uncertainty, and in the case of the initial facility size and pre-drilled wells, the decisions are costly or impossible to change. Thus, they deserve special attention. The other decision variables are either strictly constrained by the reservoir dynamics or by the aforementioned facility variables. This implies that subsequent decision makers will change the values of these variables. These implicit variables are called *endogenous variables*. The integrated asset model includes the following endogenous variables: bottomhole and injection pressures, average reservoir pressure, individual flow rates, drilling, and an expansion option.

CHAPTER FOUR: DIAGNOSTICS

Overview

This chapter presents the results of diagnostic runs performed with the IAM. Different cases were defined to test the model and to ascertain whether it would run correctly, i.e. to verify that the results generated by the model make physical sense.

Model Definition

Model One: 1 Tank, Oil Production with no Injection

This case has one compartment and involves the single phase flow of oil. Case one is set up to drill at most 4 wells throughout the field and no more than four wells can be drilled in any year. The option to expand is also allowed after three years of production. The bottom-hole pressure is constrained not to fall below 9217 psi. Case one is further sub-divided into three cases.

a. Case 1a: This case was setup with the initial facility size too big at 150,000 stb/d. By setting the initial facility size at this values it is expected that when production starts the field goes immediately into decline.

b. Case 1b: This case was setup with a moderate initial facility size of 90,000 stb/d. With this initial facility size it is expected that when production starts, a plateau period is observed before the field goes in to decline.

c. Case 1c: This case was defined with a small initial facility size 35,000 stb/d. Here it is expected that a plateau period is observed and the option to expand the facility will be exercised.

In all these cases the reservoir and fluid properties are identical and are presented in table 4 while table 5 presents the values of the parameters used in the cost function.

Table 5: Reservoir and fluid properties for one compartment and single phase flow cases

Property	**Value**
Initial pressure (psi)	20000
Porosity (fraction)	0.19
r_w (ft)	0.328
C_A	30.1
s	-0.91
k (md)	500
viscosity (cp)	1.7
c_t (psi^{-1})	2.25E-05
S_{oi}	0.79
V_p (bbls)	Uniformly distributed between 8E8 and 15E8

Table 6: Cost function parameters

Cost parameter	**Value**
Discount rate, r (%/year)	15
Well cost (US Dollars)	250E6
Facility construction coefficient one	50E6
Facility construction coefficient two	4.5E6
Facility expansion coefficient one	50E6
Facility expansion coefficient two	4.5E6
Variable production cost (US Dollars)	4.0

Expansion cost multiplier	0.75
Expansion capacity multiplier	2.0

Table 5 continued

Model One: Results and Discussion

a. Case 1a: In this case it was optimal to drill three wells in the first year and one well in the second year making a total of four wells in the first two years of the field life. This result is consistent with conventional/intuitive thinking as one would want to get as much oil as possible out of the ground as soon as possible. By constraining the code to permit at most three wells per year it is assumed that it takes approximately 120 days to drill a single well and the choice of four as the maximum number of wells that can be drilled is arbitrary (a different choice could have been made). Table 8 shows the matrix of endogenous variables indicating when a well is drilled.

Table 7: Matrix showing the endogenous decision of when and the number of wells to drill, case 1a

Wells drilled in Reservoir					
Time (Years)	**Well 1-1**	**Well 1-2**	**Well 1-3**	**Well 1-4**	**Total Per year**
1	1	0	1	1	3
2	0	1	0	0	1
Total wells drilled					4

This case was defined with the initial facility size too big so that the field goes into decline once production starts and no expansion is exercised, i.e. no plateau period and no expansion is observed. This characteristic was observed in the production profile from the field, figure 6.

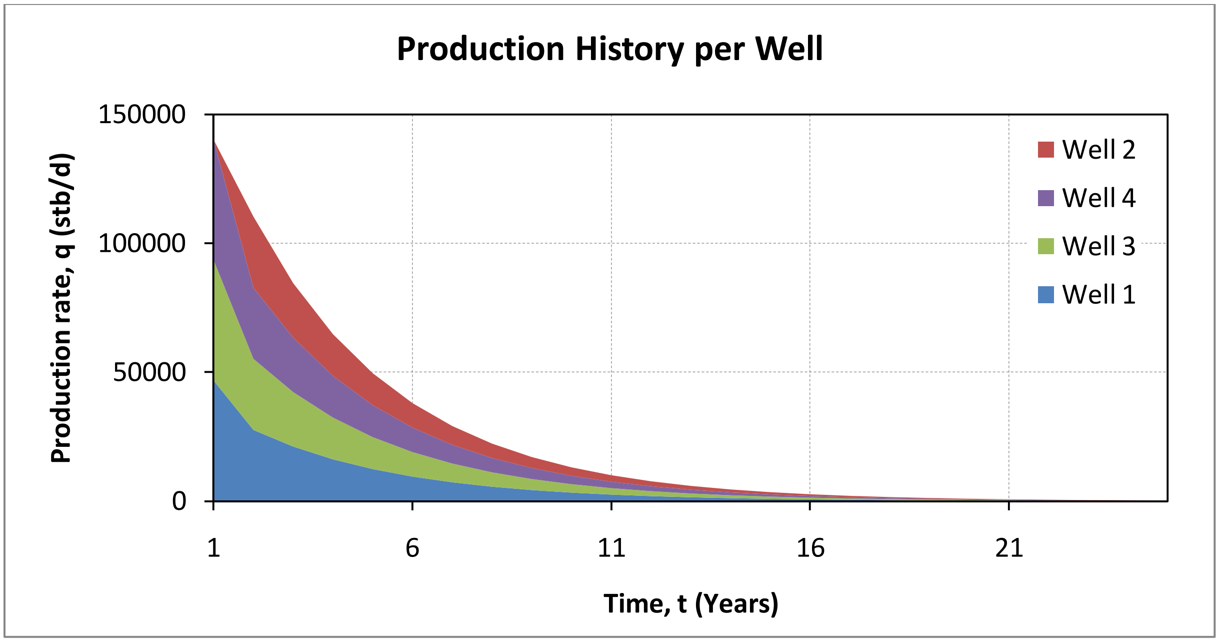

Figure 6: Production history from each well in the field plotted on a single graph, case 1a

The production from the field peaked at the end of the first year and gradually declined to approximately zero at year 25. Figure 7 presents the total production from the field and figure 8 shows the average reservoir pressure.

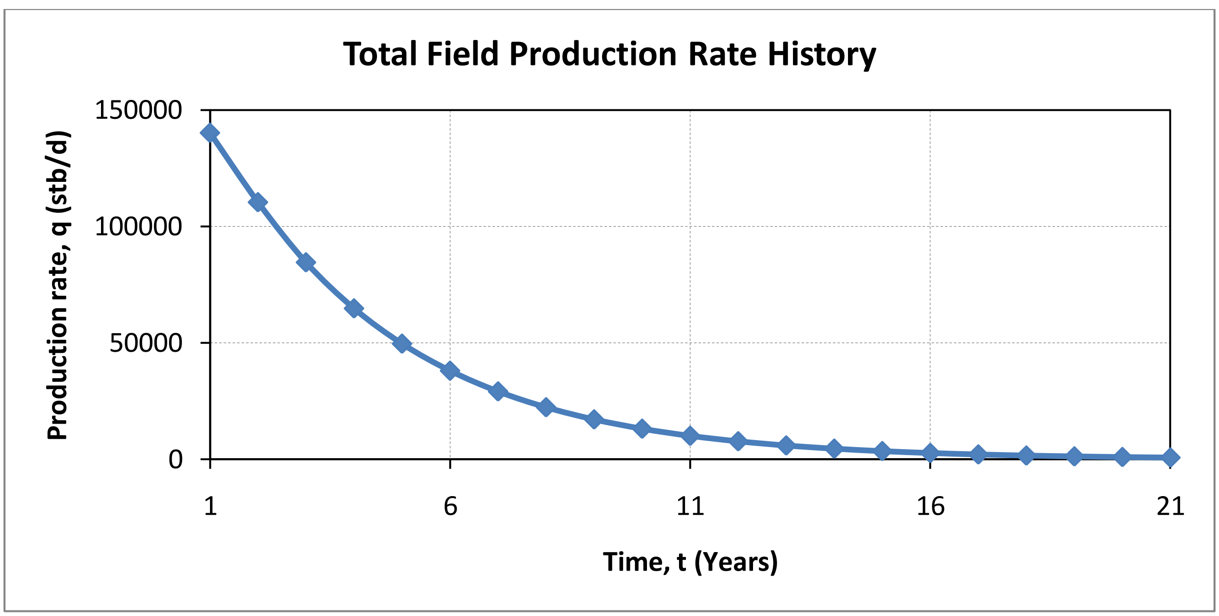

Figure 7: Total production rate from the field, case 1a

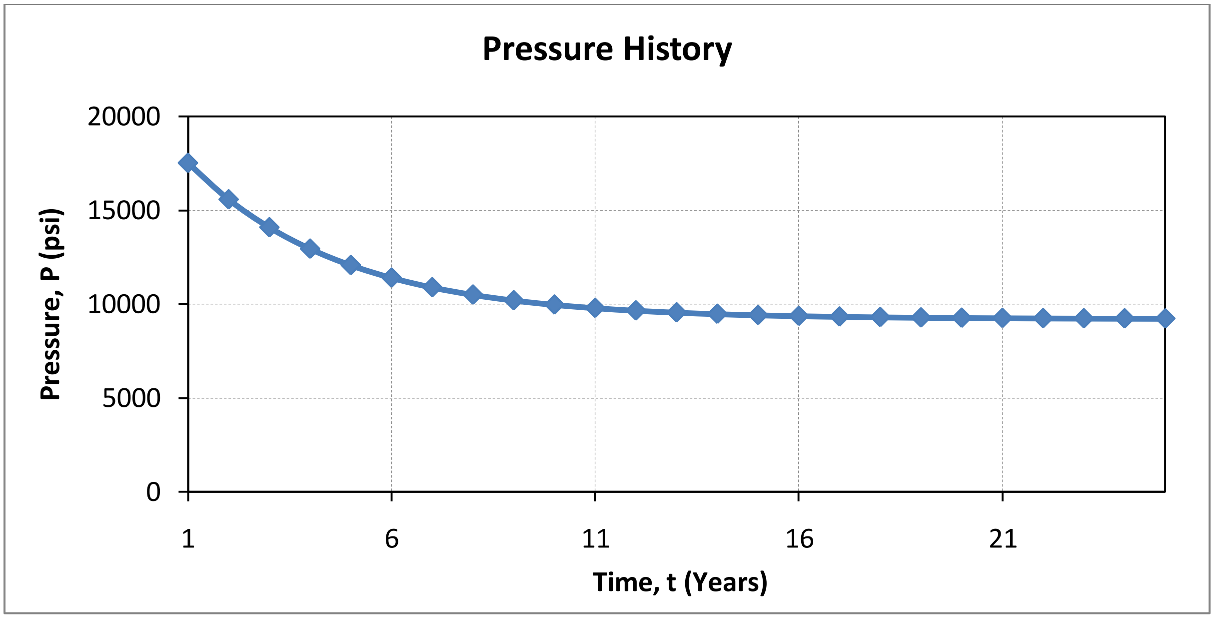

Figure 8: Field pressure history throughout production life, case 1a

From figure 8 the average reservoir pressure at the end of the first year is 17529 psi. It gradually declines from this value to about 9350 psi in year 15 and stays relatively flat until the end of the field life in year 25.

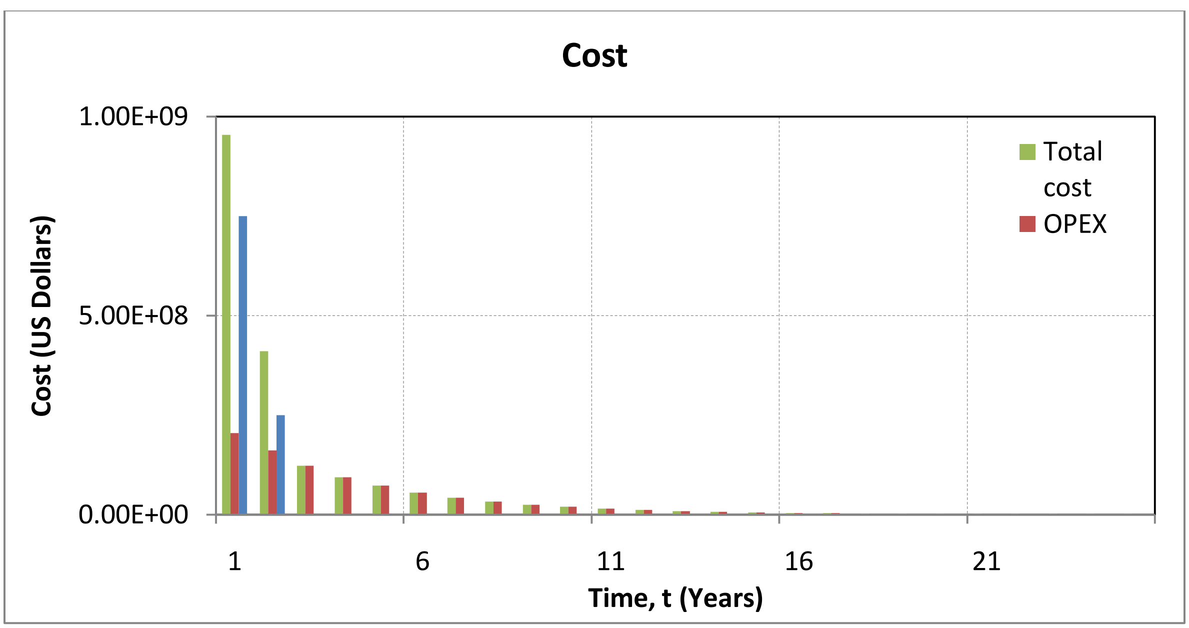

Figure 9: Cost history throughout the field life, case 1a

From figure 9 it can be seen that the capital expenditure (CAPEX) is maximum at the end of year one because of the initial facility and drilling investment. In year two, the CAPEX is less because only one well is drilled in year two. From year three to the end of the field life the CAPEX is zero. Similarly the operating expenditure (OPEX) is maximum at the end of year one and gradually decreases as the production rate decreases. The net present value (NPV) for this case was USD 5.7×10^9.

b. Case 1b: In this case three wells were drilled in the first year and the fourth well was drilled in the sixth year. This result suggests that given an initial facility size of 90,000 stb/d and a reservoir with the given properties that it is optimal to drill three wells in the first year. A fourth well does not pay for itself until the sixth year. Table 9 presents the endogenous variable indicating when wells are drilled for this case.

Table 8: Matrix showing the endogenous decision of when and the number of wells to drill, case 1b

Wells drilled in Reservoir					
Time (Years)	**Well 1**	**Well 2**	**Well 3**	**Well 4**	**Total Per year**
1	1	1	0	1	3
6	0	0	1	0	1
Total wells drilled					4

Because this case was defined with the initial facility size as moderate at 90,000 stb/d, an initial plateau period is observed between year one and two. Production from the field started declining at the end of year two.

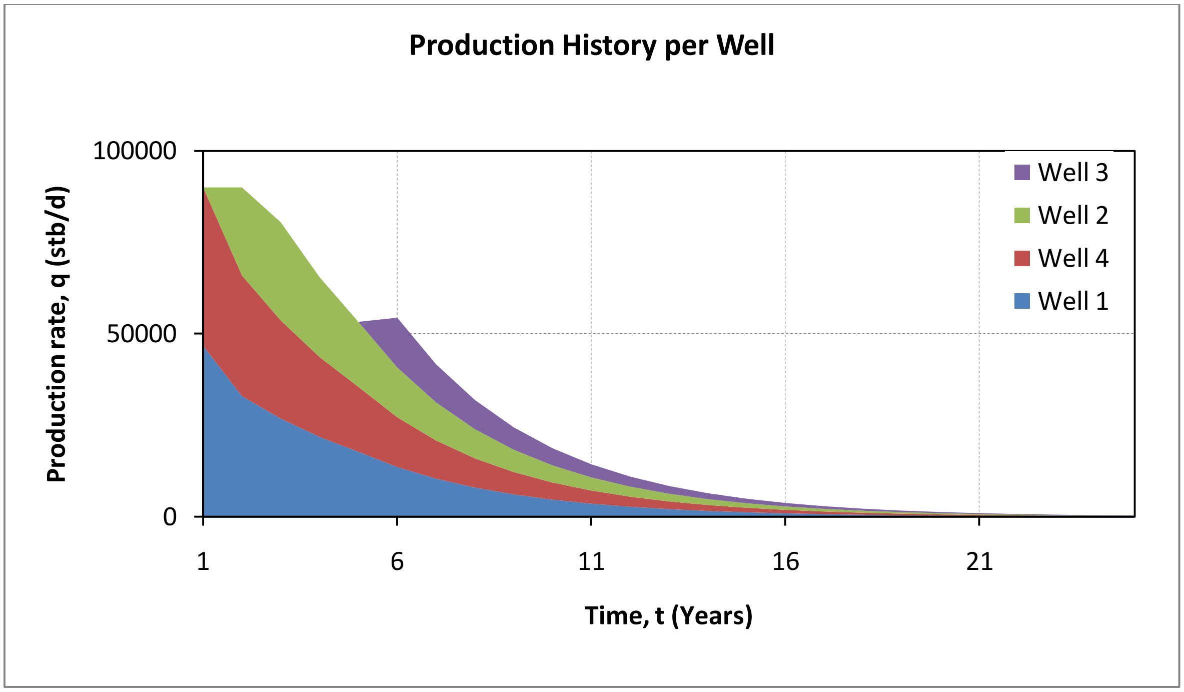

Figure 10: Production history from each well in the field plotted on a single graph, case 1b

An interesting observation from the production history is that even though three wells were drilled in year one (well 1, well 2 and well 4) well 2 did not come on stream until the end of year two indicating that it was optimal in this case to pre-drill one of the wells. Well 3 was not drilled until year six and it came on stream immediately.

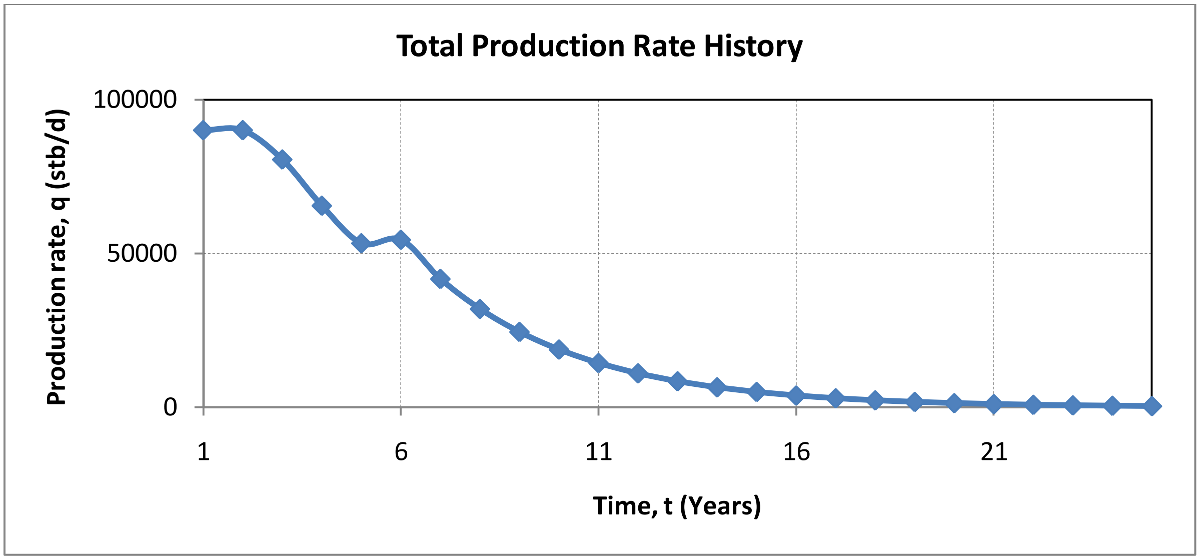

Figure 11: Total production rate from the field, case 1b

Figure 11 which depicts the production history from the field can be seen to show an initial plateau up till the end of year two after which production declines until year six. Because a new well came on stream in year six the production rate from the field increased slightly and then started to decline at the end of year six. The pressure profile in the reservoir is shown in figure 12.

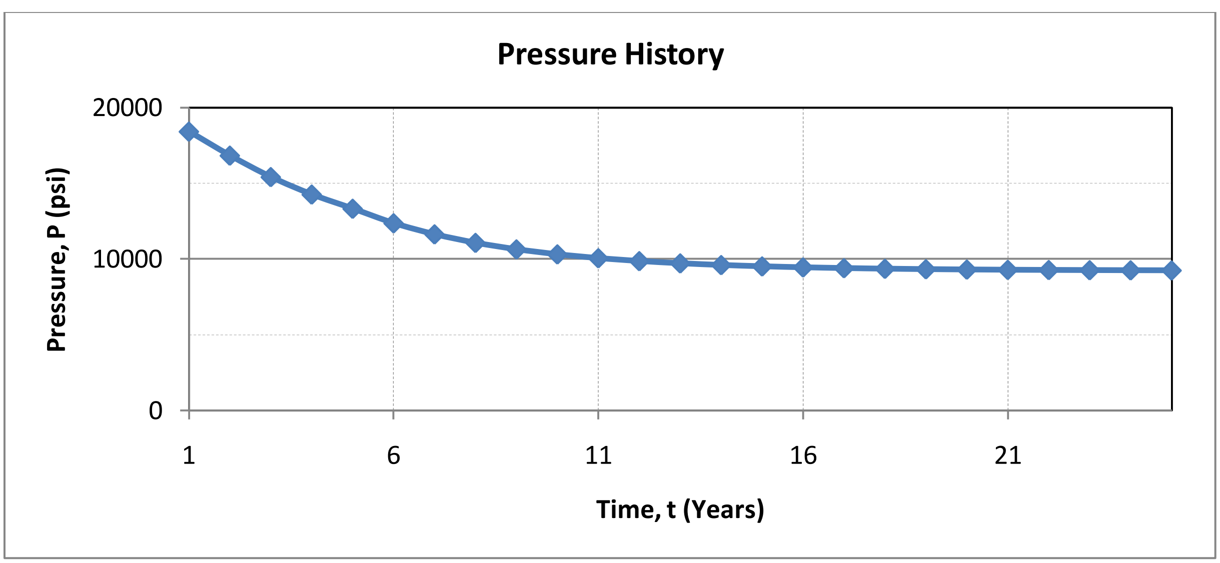

Figure 12: Field pressure history throughout production life, case 1b

From figure 12 the average reservoir pressure at the end of the first year is 18413 psi. It gradually declines from this value to about 9500 psi in year 16 and stays relatively flat until the end of the field life in year 25.

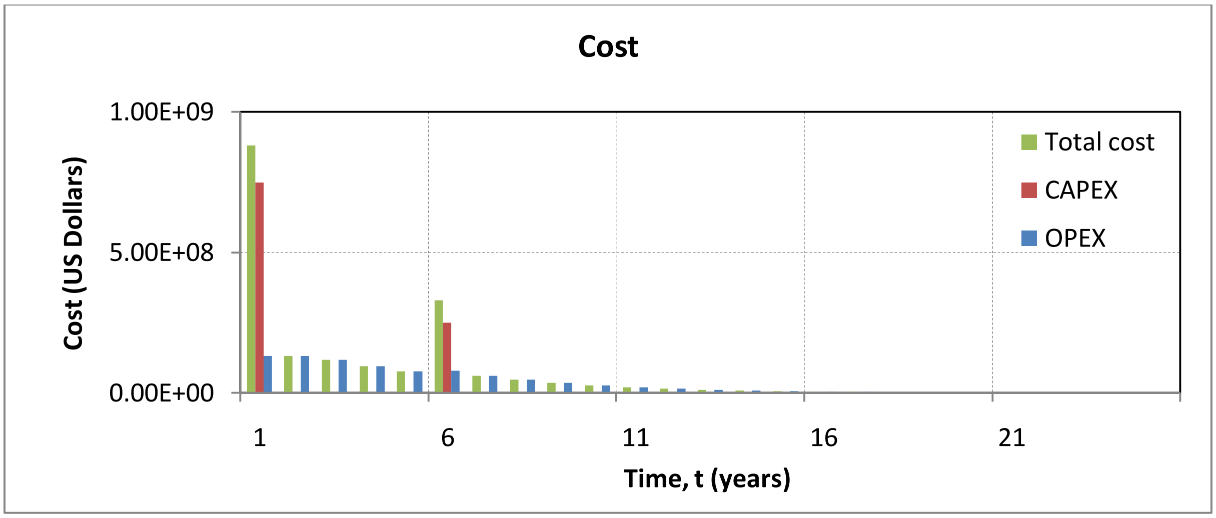

Figure 13: Cost history throughout the field life, case 1b

The cost history shows that the OPEX gradually declines from a maximum at the end of year one to a minimum at the end of the field life. The CAPEX has a maximum value in year one which accounts for the initial investment in facility size and well drilling. It is zero thereafter and only has a value in year six to account for the well drilled in that year. The NPV for this case is USD 5.3x10^9.

Case 1c: This case found it optimal to drill one well in year one, one well in year three (the expansion year) and two wells in year four. This result makes physical sense because the initial facility is small (35, 000 stb/d) and production from one well was enough to maximize the NPV for the first two years.

Table 9: Matrix showing the endogenous decision of when and the number of wells to drill, case 1c

Wells drilled in Reservoir					
Time (Years)	**Well 1**	**Well 2**	**Well 3**	**Well 4**	**Total Per year**
1	0	1	0	0	1
3	0	0	0	1	1
4	1	0	1	0	2
Total wells drilled					4

Because the expansion option was exercised in year three another well was drilled and two additional wells were drilled in year four. Figure 14 shows the contribution of each well to the total production from the field. It is interesting to see how production from each well balanced to optimally utilize the facility capacity. Well 2 (drilled in year one) was shut-in in year four as well 3 (drilled in year four) came on stream. At the end of year four well 3 was shut-in and well 2 came back on stream. Well 3 was later open to flow at the end of year six.

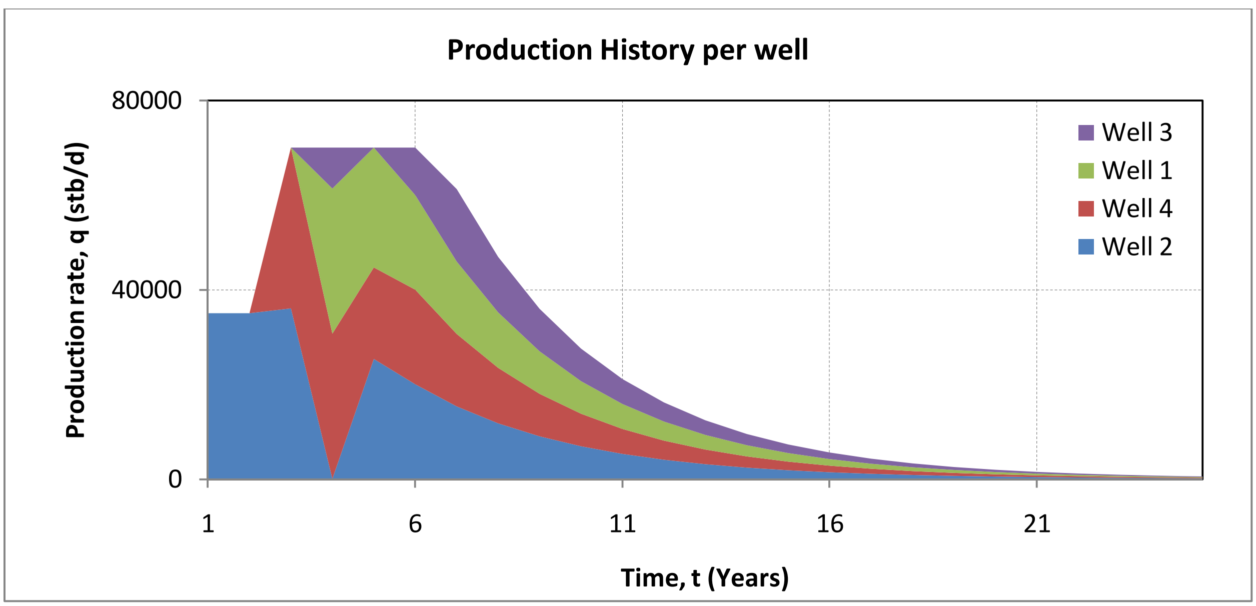

Figure 14: Production history from each well in the field plotted on a single graph, case 1c

The pressure profile for this case is presented in the figure 15 below.

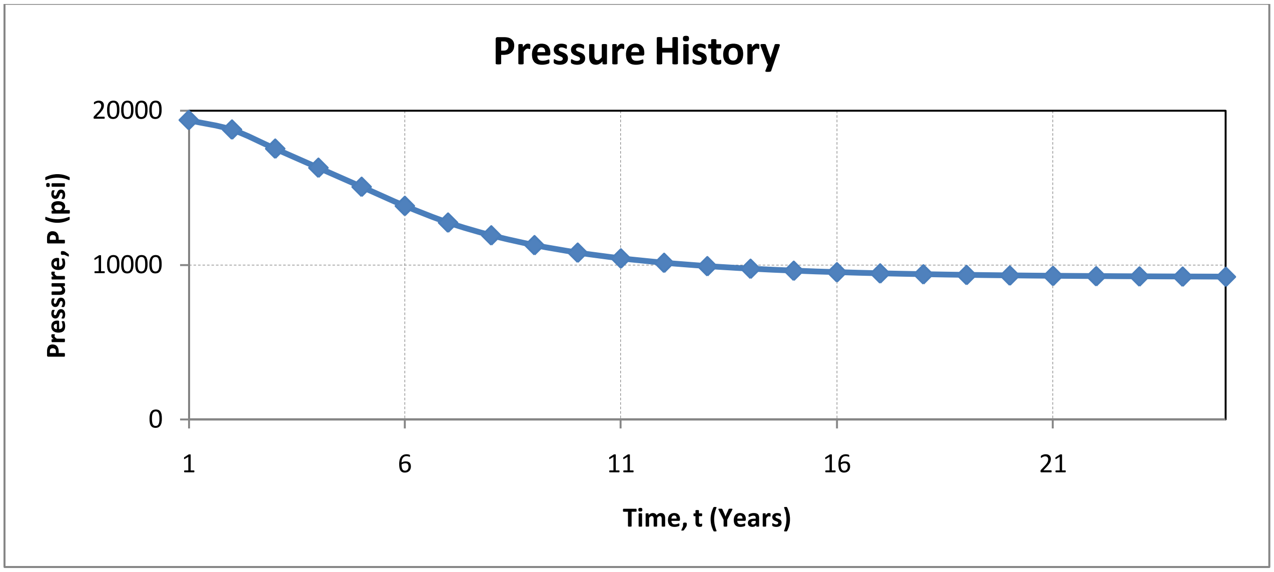

Figure 15: Field pressure history, case 1c

The cost history in this case is also consistent with cost being maximum when major facility investment are made at the early stages of the field life and minimum at the end (figure 16). The NPV in this case is USD 4.4x10^9.

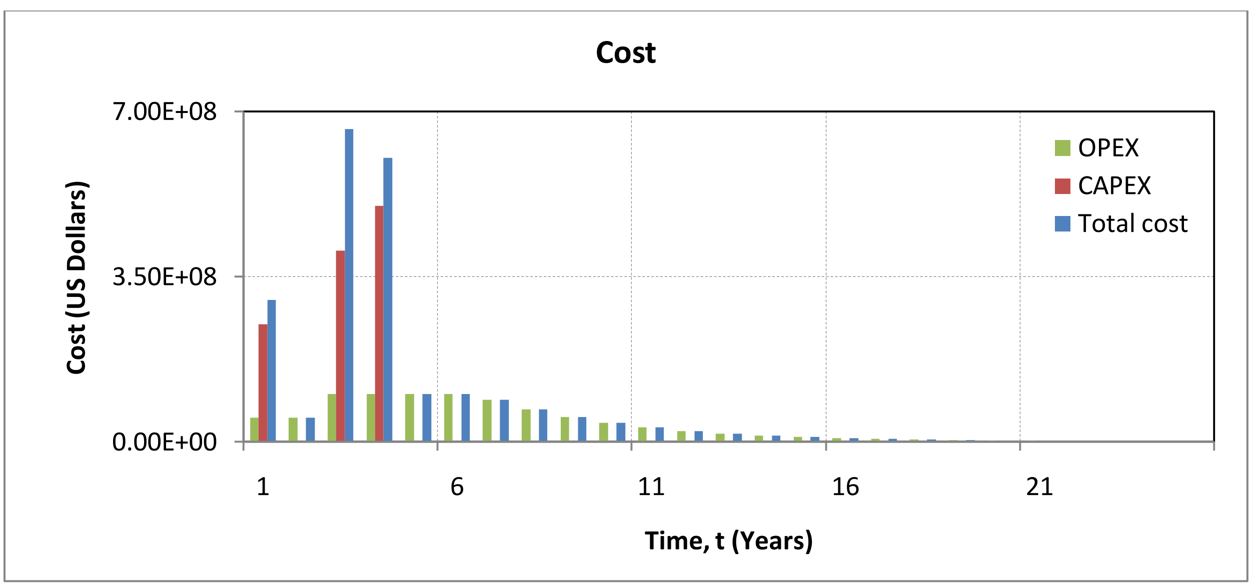

Figure 16: Cost history throughout the field life, case 1c

Model Two: 2 Tanks with oil Production

The purpose of this model is to capture heterogeneity within the reservoir by dividing the reservoir into 2 compartments. The underlying assumption here is that the reservoir is heterogeneous and has two permeability distributions. Compartment 1 has a high permeability which is representative of the expected value of one of the PDFs describing the reservoir's permeability distribution while compartment 2 has a low permeability that represents the expected value of the second PDF describing the reservoir's permeability. Both compartments produce through the same facility.

For demonstration purposes, this model can drill at most 10 wells in each compartment (a total of 20 wells) and no more than four wells can be drilled in any particular year. The option to expand is also allowed after four years of production. The bottom-hole pressure is constrained to be greater than or equal to 9217 psi.

Three demonstration cases are run:

a. Case 2a: This case was setup with the initial facility size too big at 500,000 stb/d. With this initial facility size it is expected that when production starts the field will immediately go into decline.

b. Case 2b: This case was setup with a moderate initial facility size 200,000 stb/d. With this initial facility size it is expected that when production starts a plateau period is observed before the field goes in to decline.

c. Case 2c: This case was defined with a small initial facility size 100,000 stb/d. Here it is expected that a plateau period is observed and the option to expand the facility will be exercised.

The input reservoir and fluid (oil) properties for this model are presented in table 6. The parameters in the cost functions are as shown in table 2.

Table 10: Input reservoir and fluid properties for all cases defined for model two

Single Phase Flow		
Property	**Compartment 1**	**Compartment 2**
Initial pressure (psi)	20000	20000
Porosity (fraction)	0.19	0.15
r_w (ft)	0.328	0.328
C_A	30.1	30.1
s	-0.91	-0.91
k (md)	150	500
viscosity (cp)	1.7	1.7
C_t (psi^{-1})	2.25E-05	2.25E-05
Transmissibility (bbls/d/psi)	20	
S_{oi}	0.79	0.69
V_p (bbls)	Uniformly distributed between 100E7 and 150E7	Uniformly distributed between 80E7 and 120E7

Model Two: Results and Discussion

Case 2a: A total of nine wells were drilled in this case out of which seven were

Table 11: Matrix showing the endogenous decision of when and the number of wells to drill, case 2a

Wells drilled in Reservoir (Compartment 1)									
Time (Years)	Well 1-1	Well 1-2	Well 1-3	Well 1-4	Well 1-5	Well 1-6	Well 1-7	Well 1-8	Total Per year
1	0	0	0	0	1	1	0	0	2
2	1	1	1	0	0	0	0	0	3
3	0	0	0	0	0	0	0	1	1
4	0	0	1	0	0	0	0	0	1
Total wells drilled in compartment 1									7
Wells drilled in Reservoir (Compartment 2)									
Time (Years)	Well 2-1	Well 2-2	Well 2-3	Well 2-4	Well 2-5	Well 2-6	Well 2-7	Well 2-8	Total Per year
1	0	0	0	0	1	0	0	0	1
2	0	0	0	0	0	0	0	1	1
Total wells drilled in compartment 2									2
Total wells drilled in the field									9

drilled in compartment 1 and the remaining two wells were drilled in compartment 2. It is noted that two wells were drilled in compartment 1 in year one and three wells were drilled in year 2 and one well in year 3. In compartment 2, one well was drilled in year 1 and another well in year 2, making a total of two wells drilled in compartment 2. Because compartment 2 is a better reservoir sand than compartment 1 (k2 > k1) the model drilled a smaller number of wells in compartment 2. Compartment 1 required more wells to drain it because the quality of the reservoir sand was inferior to that of compartment 2.

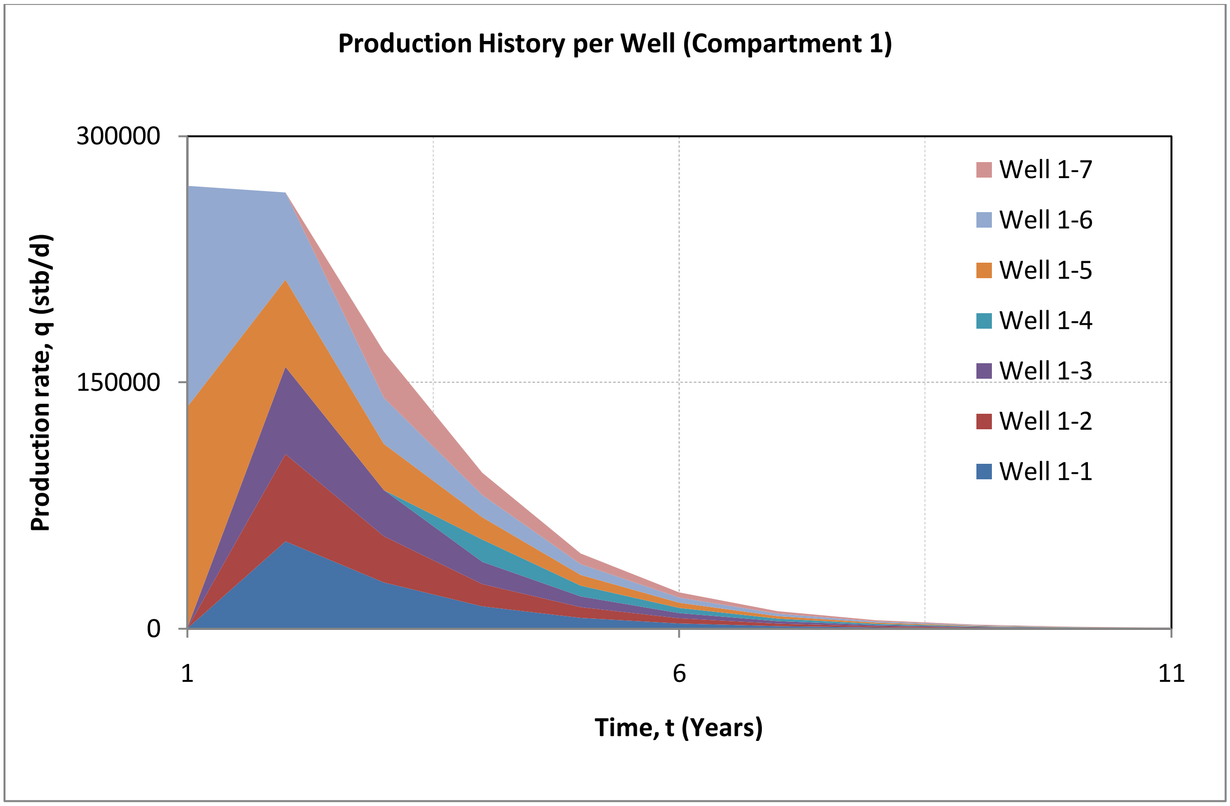

Figure 17: Production history from each well in compartment 1 plotted on a single graph, case 2a

Figure 17 and 18 show the production history from each well drilled in the field for each compartment respectively. Production from compartment 1 peaked at the end of year one and quickly declined until production was almost negligible in year eight. In compartment 2, production was also at a maximum in the first year and rapidly declined until year three when production is almost negligible.

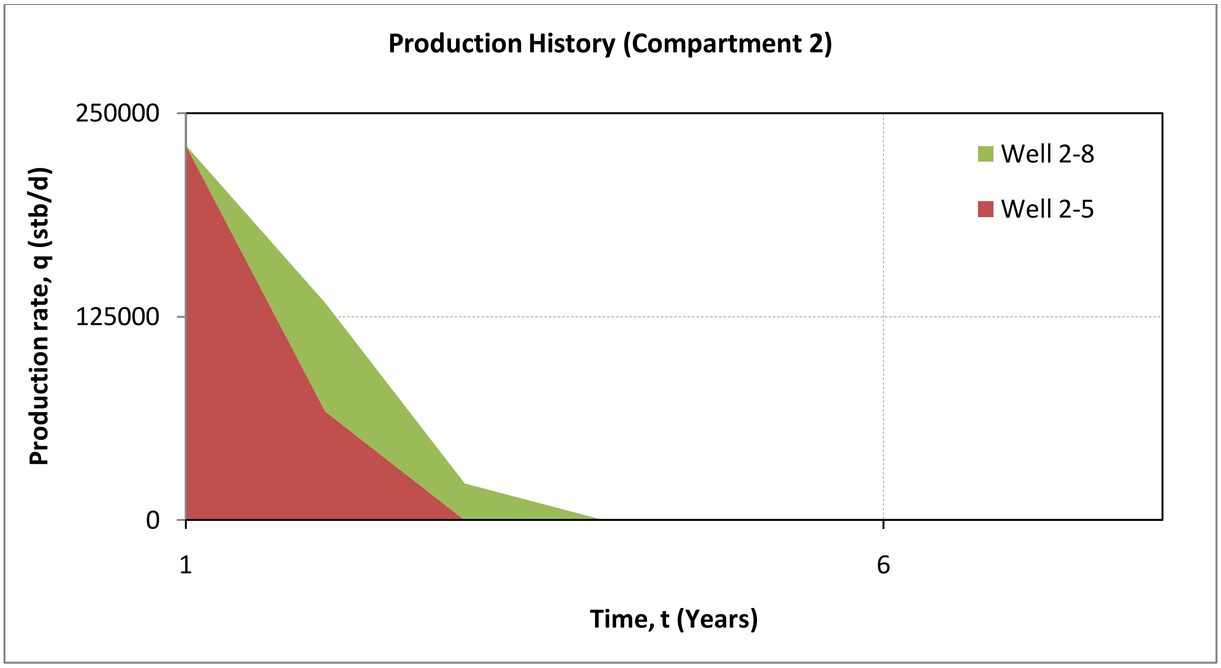

Figure 18: Production history from each well in compartment 2 plotted on a single graph, case 2a

Figure 19: Total production from the field, case 2a

As seen from figure 19 the total production from the field did not see a plateau period and this is because the initial facility size was set to a large value. The pressure profile in each compartment is shown in figure 20 below.

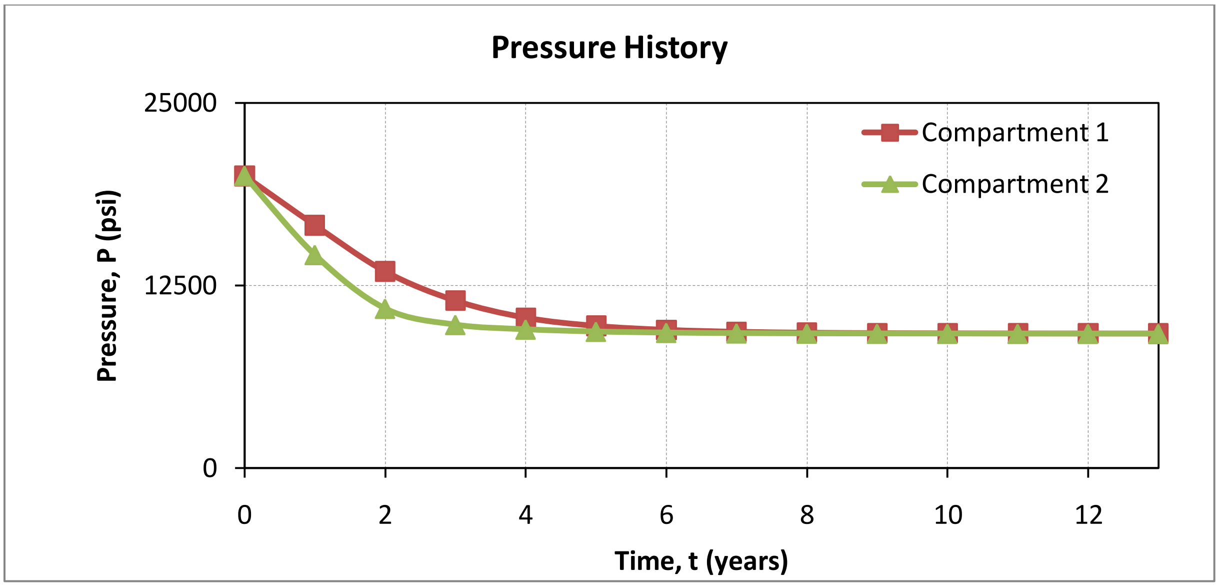

Figure 20: Pressure history for compartment 1 and 2, case 2a

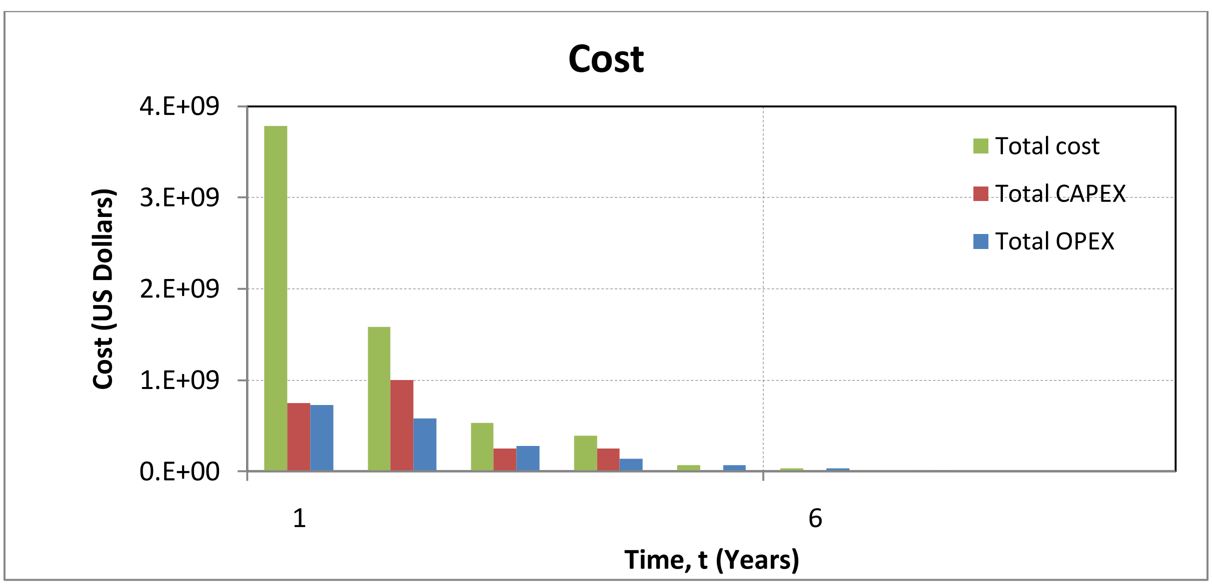

Figure 21: Cost history, case 2a

Case 2b: A total of 2 wells were drilled in the field, one in each compartment

Table 12: Matrix of endogenous variable showing when and how many wells drilled

Wells drilled in Reservoir (Compartment 1)									
Time (Years)	Well 1-1	Well 1-2	Well 1-3	Well 1-4	Well 1-5	Well 1-6	Well 1-7	Well 1-8	Total Per year
1	0	1	0	0	0	0	0	0	1
Total wells drilled in compartment 1									1
Wells drilled in Reservoir (Compartment 2)									
Time (Years)	Well 2-1	Well 2-2	Well 2-3	Well 2-4	Well 2-5	Well 2-6	Well 2-7	Well 2-8	Total Per year
1	1	0	0	0	0	0	0	0	1
Total wells drilled in compartment 2									1
Total wells drilled in the field									2

The production profile from each well in compartment 1 is shown in figure 22 while the production profile from compartment 2 is shown in figure 23.

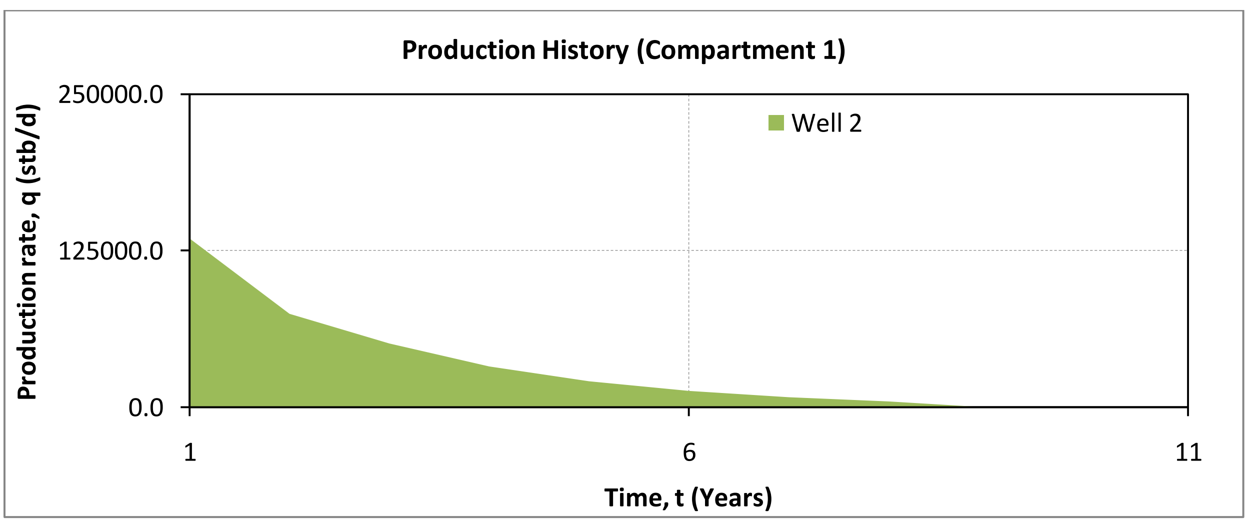

Figure 22: Production history from each well in compartment 1, case 2b

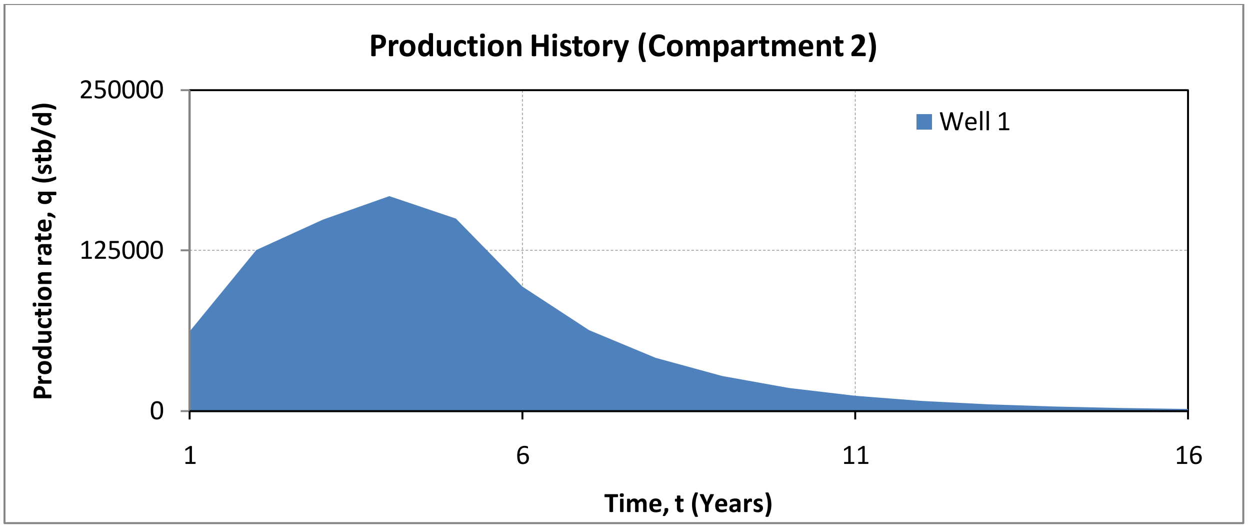

Figure 23: Production history from each well in compartment 2, case 2b

The total production rate from the field is presented in figure 24. The production profile for the field showed a plateau of 200,000 stb/d for the first 4 years after which the expansion option was exercised. After the expansion in year four the field production went in to decline and was almost zero in year 16.

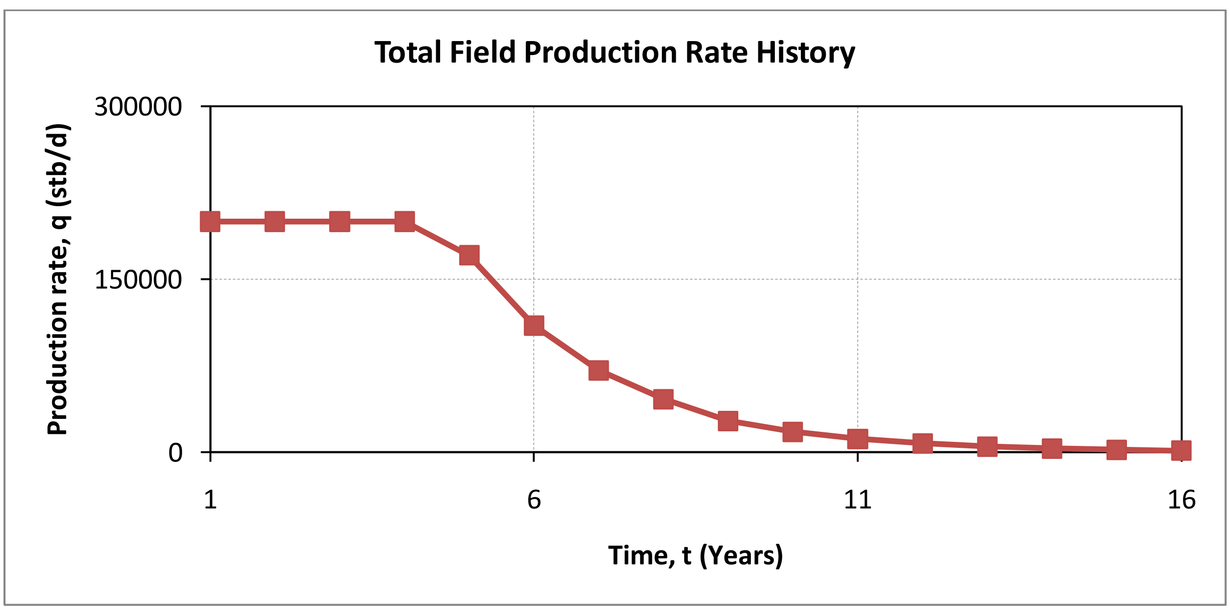

Figure 24: Total field production history, case 2b

The pressure profile in each compartment is shown on a single plot in figure 25.

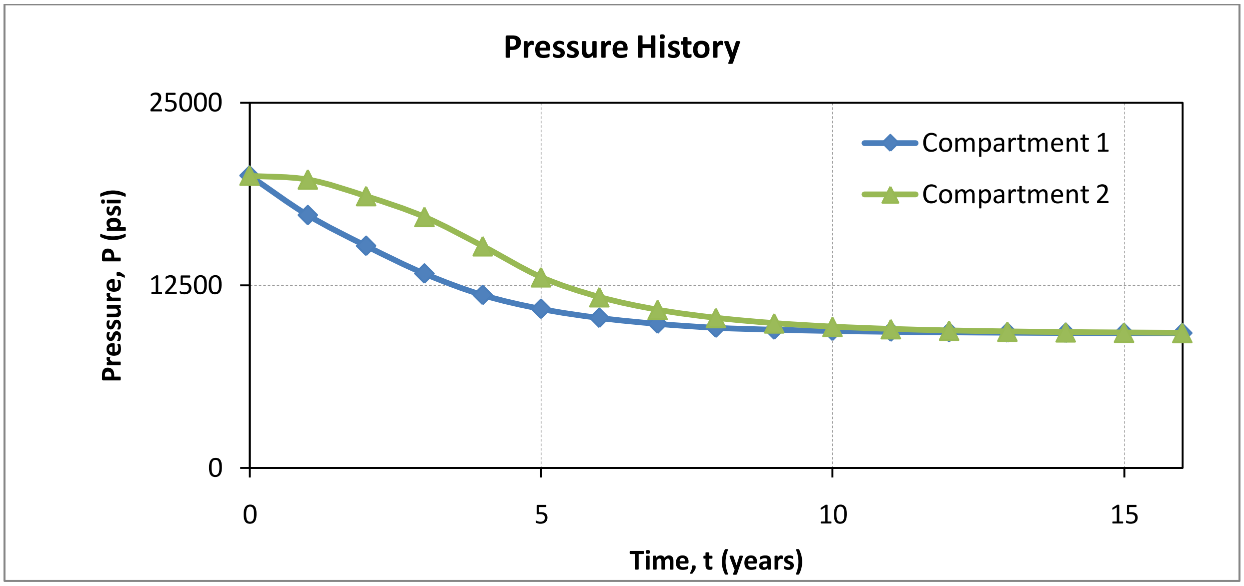

Figure 25: Pressure history from each compartment in the field, case 2b

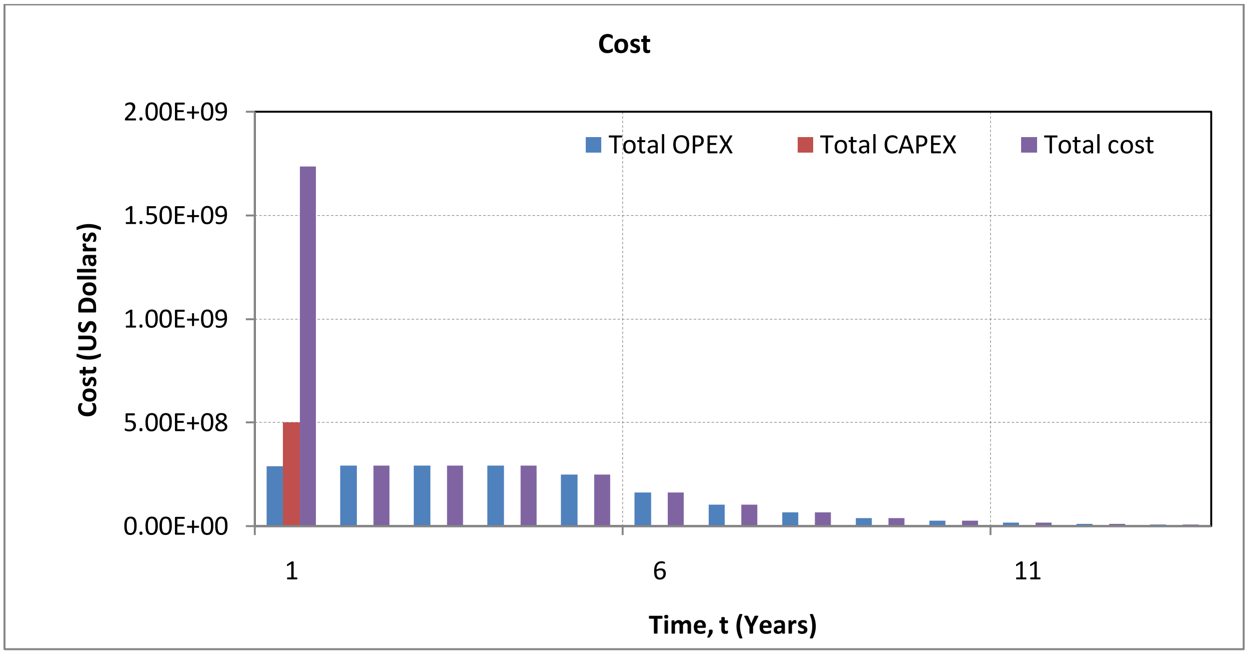

Figure 26: Cost history, case 2b

The cost history for this case is shown in figure 26 and the NPV was USD $1.5X10^{10}$.

Case 2c: Four wells were drilled in compartment 1 and two wells were drilled in compartment 2, a total of six wells were drilled in the field.

Table 13: Matrix of endogenous variable showing when and how many wells drilled

Wells drilled in Reservoir (Compartment 1)									
Time (Years)	Well 1-1	Well 1-2	Well 1-3	Well 1-4	Well 1-5	Well 1-6	Well 1-7	Well 1-8	Total Per year
1	0	0	0	0	0	0	1	0	1
9	0	0	0	0	1	0	0	0	1
10	0	1	0	0	0	0	0	1	2
Total wells drilled in compartment 1									4
Wells drilled in Reservoir (Compartment 2)									
Time (Years)	Well 2-1	Well 2-2	Well 2-3	Well 2-4	Well 2-5	Well 2-6	Well 2-7	Well 2-8	Total Per year
2	0	0	0	0	0	1	0	0	1
12	0	0	0	0	0	0	0	1	1
Total wells drilled in compartment 2									2
Total wells drilled in the field									6

The production history from each well in compartment 1 is shown in figure 27 while the production history from each well in compartment 2 is shown in figure 28.

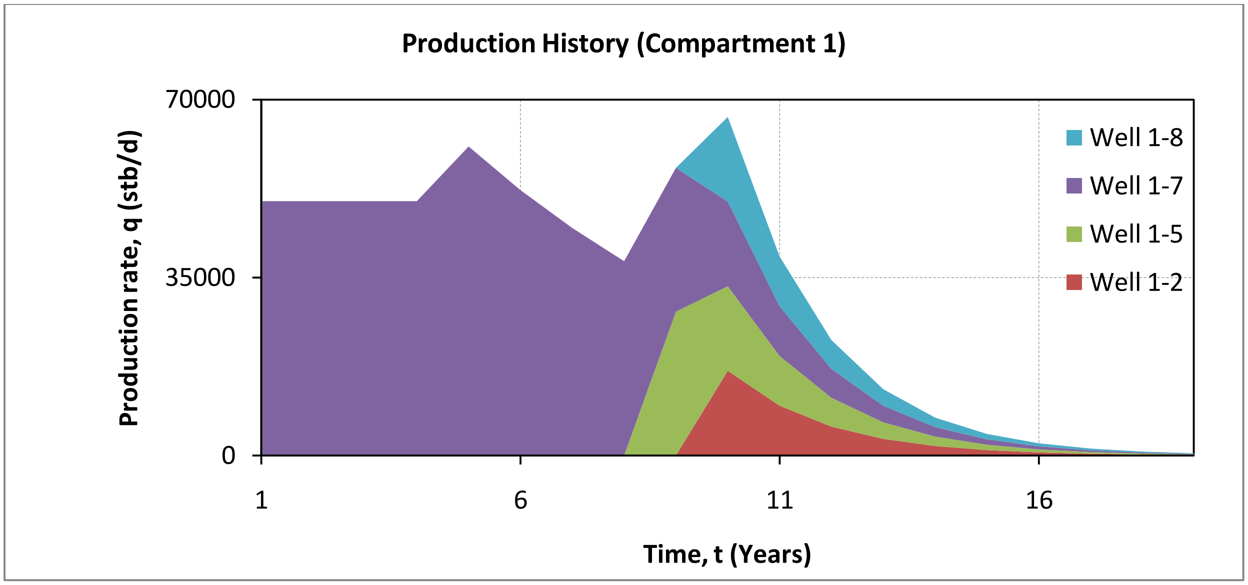

Figure 27: Production history from each well in compartment 1, case 2c

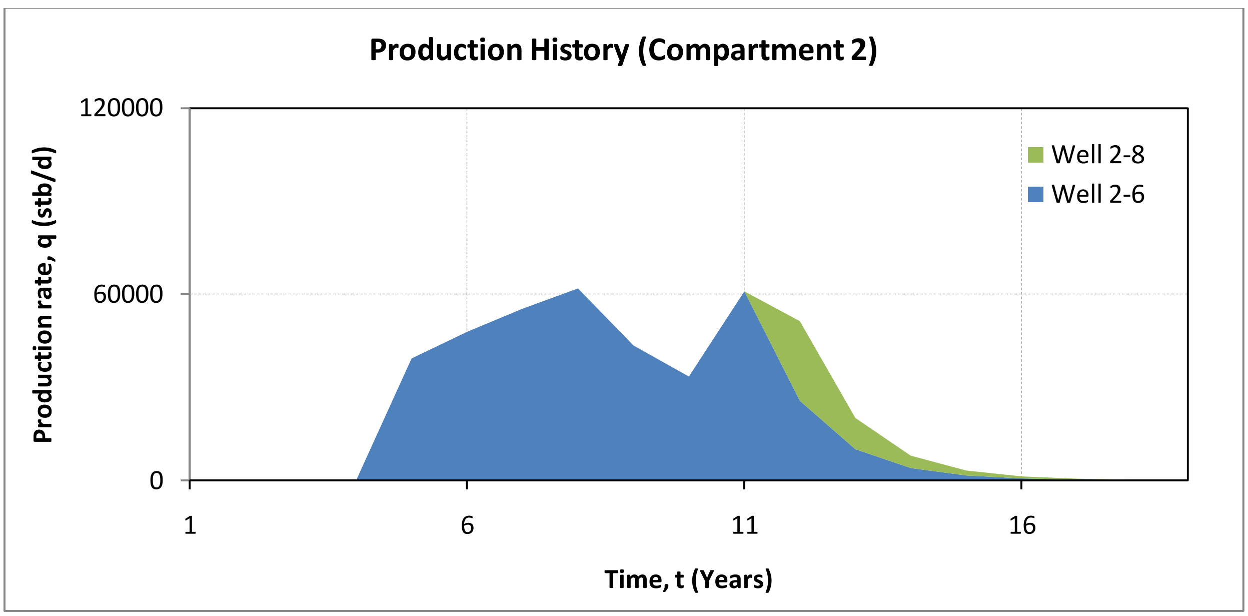

Figure 28: Production history from each well in compartment 2, case 2c

The total production rate from the field is presented in figure 29. The production profile for the field showed a plateau of about 50,000 stb/d for the first four years after which the expansion option was exercised. After the facility expansion a second plateau period is observed for about seven years before the field went in to decline.

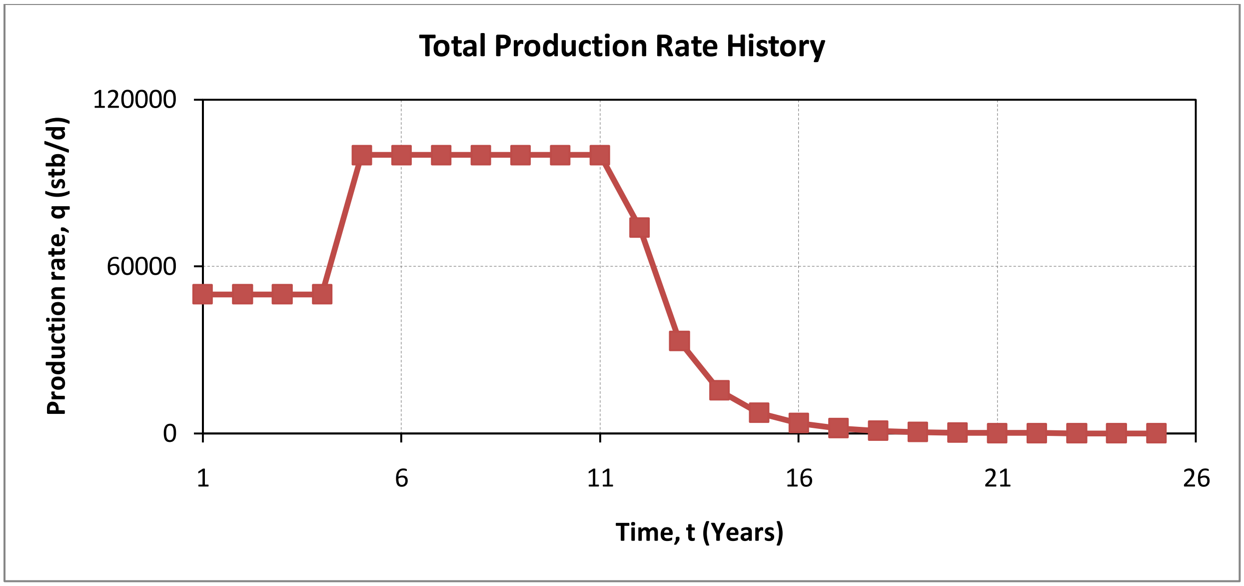

Figure 29: Total field production history, case 2c

The pressure history in each compartment is shown on a single plot in figure 30.

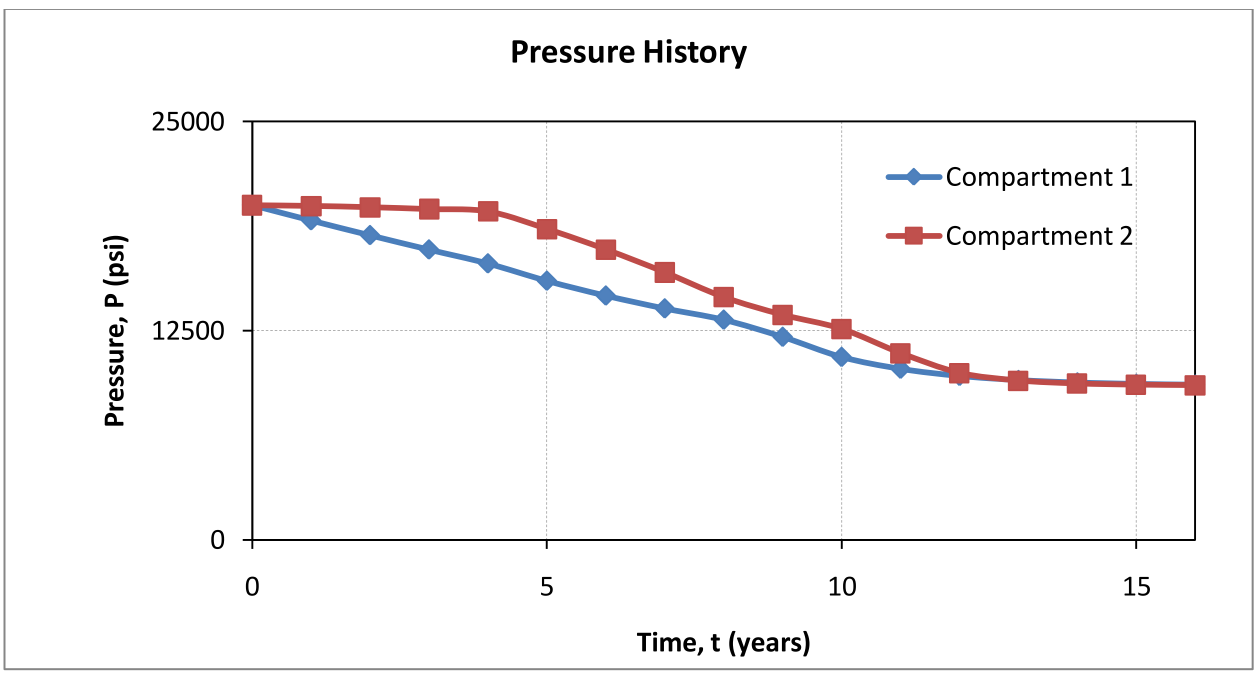

Figure 30: Pressure history from each compartment in the field, case 2c

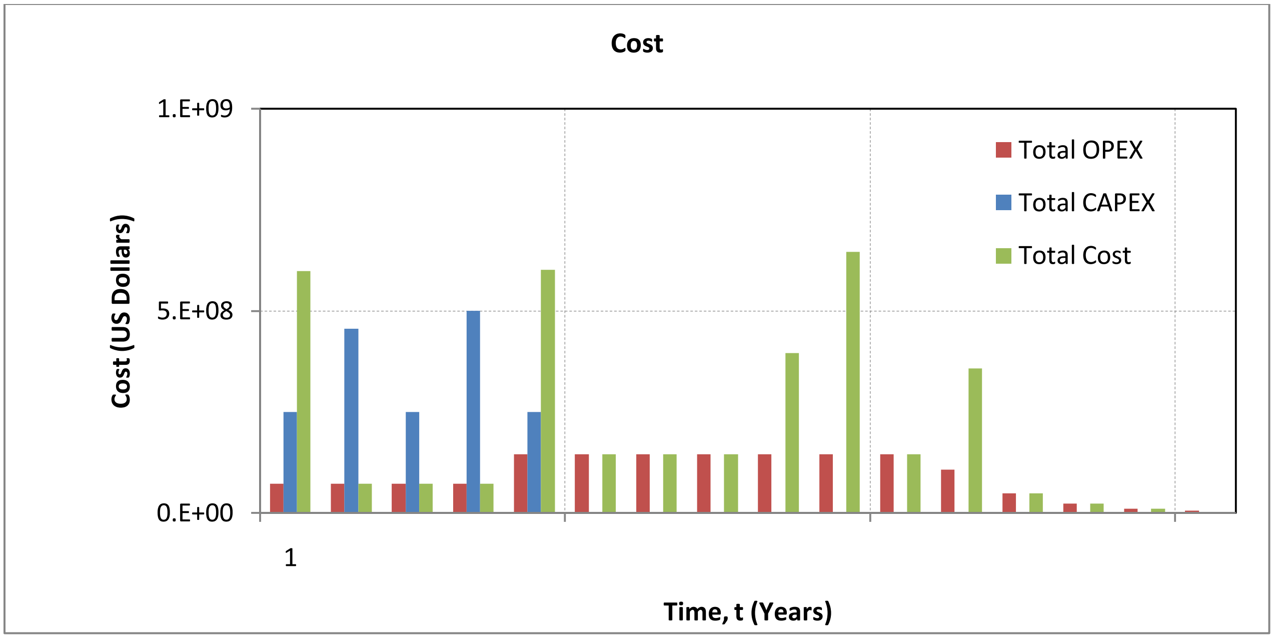

Figure 31: Cost history, case 2c

The cost history for this case is shown in figure 31 and the NPV was USD 1.2×10^{10}.

Model Two (special case): 2 Tanks with oil Production

This model is identical to model two and only differs from it because the 2 tanks are not communicating with each other. Such a situation can arise in the reservoir when a sealing fault or an unconformity divides a reservoir into two non-communicating compartments. The reservoirs should still be modeled together because production occurs in a shared facility, and other constraints (e.g. annual drilling constraint) apply to the whole project. The diagnostic case defined for this model is identical to case 1a defined for model two above except that the initial facility capacity is set equal to 200000 bbl/d. The input reservoir parameters are presented in table 13 and cost parameters are identical to those presented at the beginning of this chapter.

Table 14: Reservoir parameters for the special case of model 2

Single Phase Flow		
Property	**Compartment 1**	**Compartment 2**
Initial pressure (psi)	20000	20000
Porosity	0.19	0.15
r_w (ft)	0.328	0.328
C_A	30.1	30.1
S	-0.91	-0.91
K (md)	8	4
viscosity (cp)	1.7	1.7
C_t (psi^{-1})	2.25E-05	2.25E-05
Transmissibility (bbls/d/psi)	0	
S_{oi}	0.79	0.69
V_p (bbls)	2.2E9	1.7E9

Model Two (special case): Results and Discussion

Case 2a SP:

Table 15: Matrix of endogenous variable showing when and how many wells drilled in each compartment of the reservoir

Wells drilled in Reservoir (Compartment 1)											
Time (Years)	Well 1-1	Well 1-2	Well 1-3	Well 1-4	Well 1-5	Well 1-6	Well 1-7	Well 1-8	Well 1-9	Well 1-10	Total Per year
1	1	0	1	0	0	1	0	0	1	0	4
2	0	1	0	0	0	0	1	1	0	1	4
3	0	0	0	1	1	0	0	0	0	0	2
Total wells drilled in compartment 1											10
Wells drilled in Reservoir (Compartment 2)											
Time (Years)	Well 2-1	Well 2-2	Well 2-3	Well 2-4	Well 2-5	Well 2-6	Well 2-7	Well 2-8	Well 2-9	Well 2-10	Total Per year
3			1		1						2
4		1		1			1	1			4
5	1					1			1	1	4
Total wells drilled in compartment 2											10
Total wells drilled in the field											20

A total of 20 wells were drilled in this case, 10 in each compartment. Looking at the result in table 14, the wells in compartment 1 were drilled before the wells in compartment 2. This occurred because compartment 1 has a higher permeability than compartment 2.

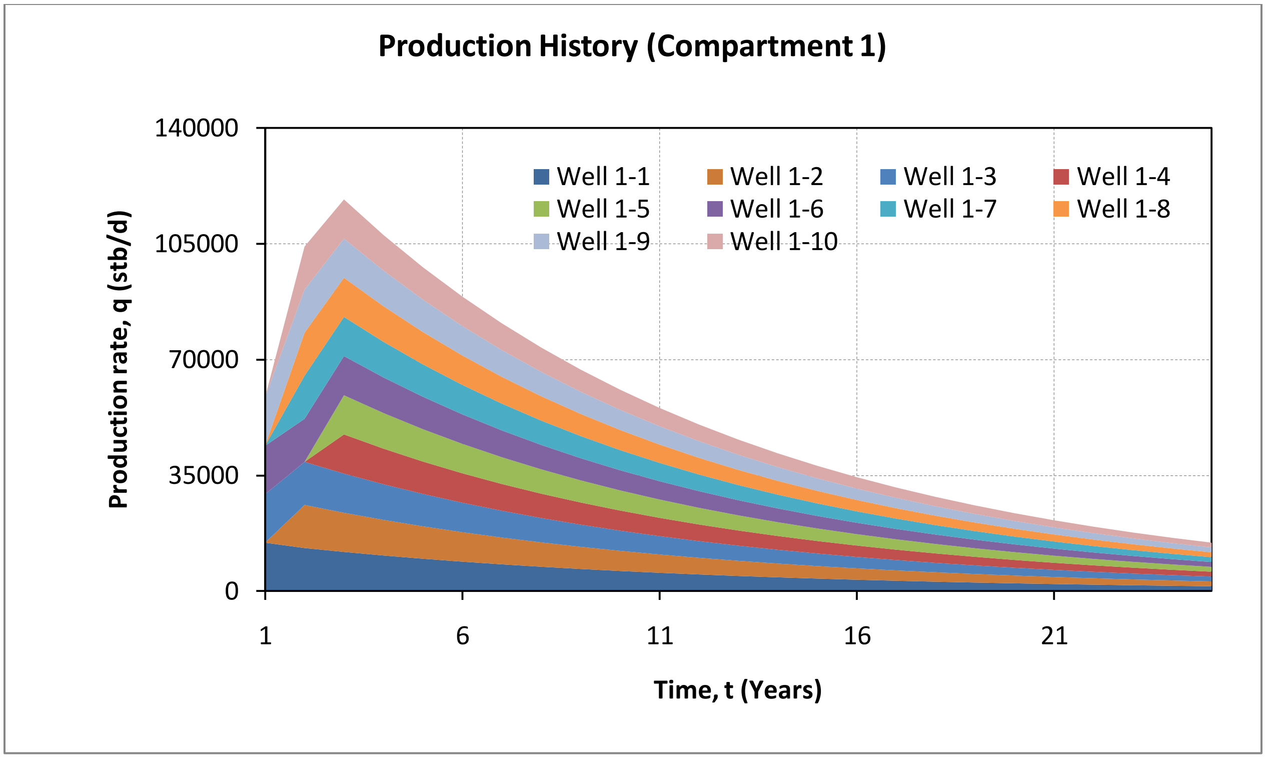

Figure 32: Production history from each well in compartment 1, case 2a SP

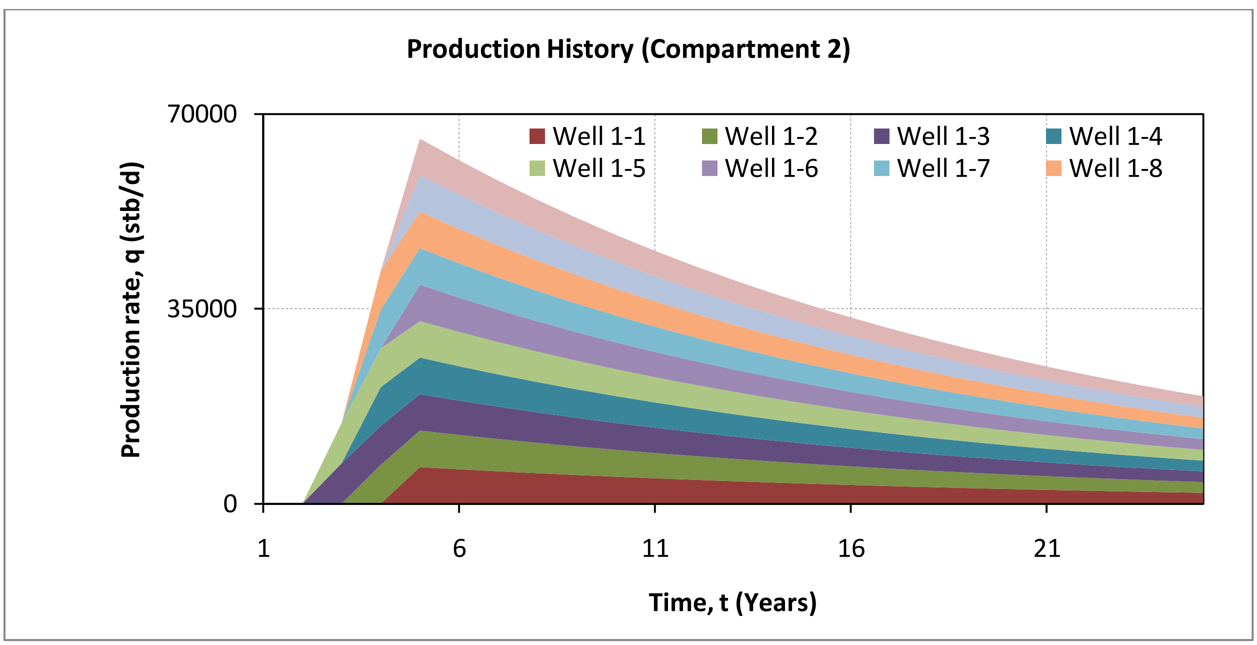

Figure 33: Production history from each well in compartment 2, case 2a SP

Figure 32 and 33 depicts the production history from each well in each compartment of the reservoir. The total production rate from the reservoir is presented in figure 34.

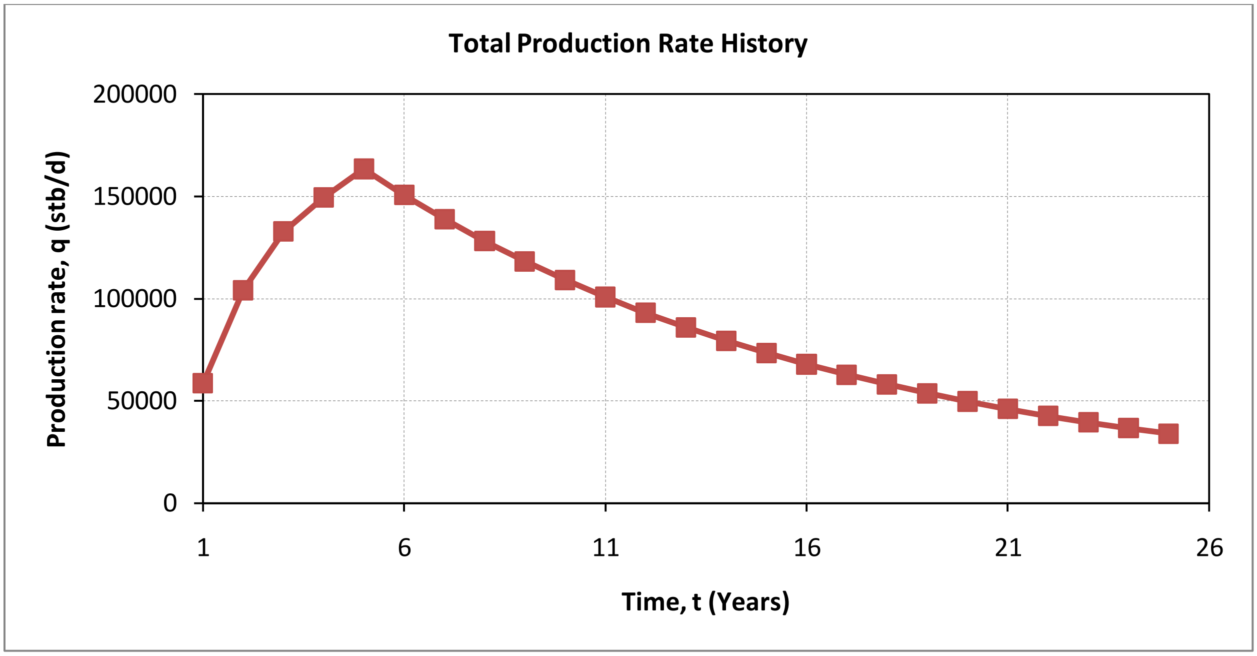

Figure 34: Total field production history, case 2a SP

From figure 34 it observed that the total production from the field gradually increased and peaked in year 5 at around 160000 bbl/d and started to decline afterward. The pressure profile is presented in figure 35 below. The pressure profile clearly shows that the compartments are not in communication.

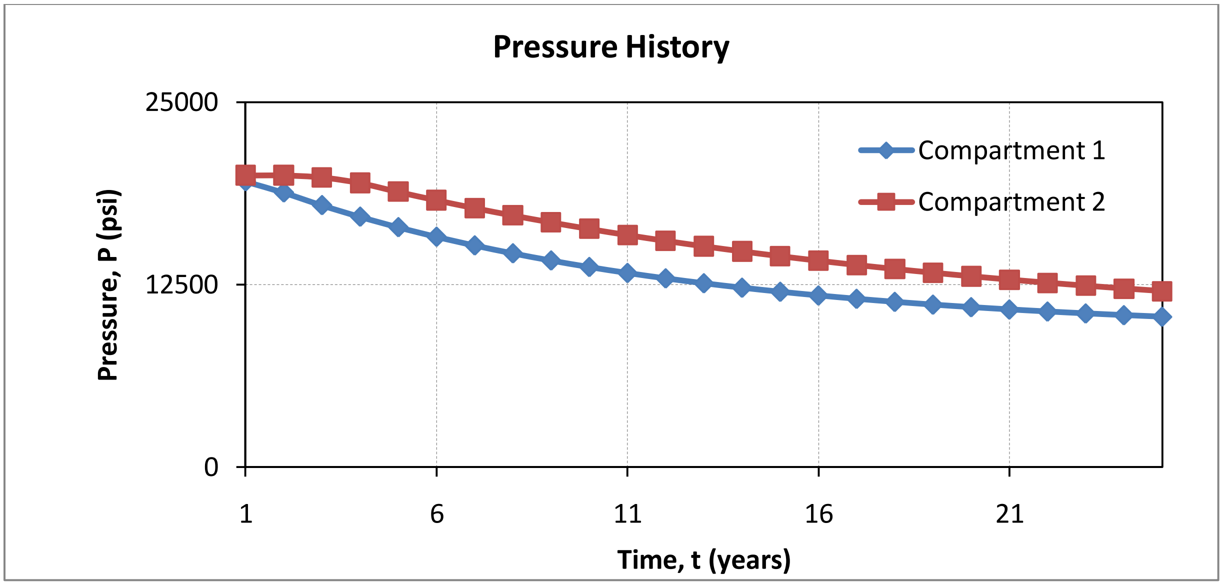

Figure 35: Pressure history from each compartment in the field, case 2a, SP

CHAPTER FIVE: OPTIMIZATION UNDER UNCERTAINTY

Introduction

In this chapter, the previously defined models are used to analyze three major design decisions for a hypothetical deepwater project. The three decisions are: (A) optimal initial production capacity, (B) optimal number of predrilled wells, and (C) whether to employ one or two rigs during the development drilling program. The challenge for project planners in these decision settings is that important inputs into the analysis are not known with certainty. Rather, some inputs are defined as random variables.

This chapter defines an uncertainty analysis workflow and explains how the IAM is used within that workflow to investigate the impact of uncertainty on the optima. The three uncertainties that are investigated are: (i) net to gross ratio (NTG) and (ii) reservoir continuity (defined by transmissibility factors between tanks).

Uncertainty Analysis Workflow

The general procedure for performing the uncertainty analysis is as follows:

1. Identify the decisions to be analyzed. For example, one decision is the initial production capacity. Different options are defined and evaluated individually.
2. Define the uncertain variables as probability distribution functions (PDFs)
3. Run the IAM within the uncertainty analysis workflow. To evaluate each decision option, the model samples the PDF(s) to obtain realizations of the uncertain variable which is then used by the model in a deterministic manner to obtain the maximum NPV (given the initial conditions and other constraints). The uncertain variable is then re-sampled and the

maximization is repeated. The result is a distribution of NPV for the specified decision option. The number of iterations used in this analysis is 25.

4. Other decision options are then specified and the process in step 3 is repeated. Thus, what results is a suite of distributions representing different options.

5. Analyze the result. Because the results for each decision option are distributions, the decision-maker can evaluate the mean, variance, and shape of the distribution. For risk-neutral decision-makers, the best option is the one with the largest NPV. If other risk preferences are in play, then the decision-maker will make a decisions based on his utility function. In this thesis we assume a risk-neutral decision-maker.

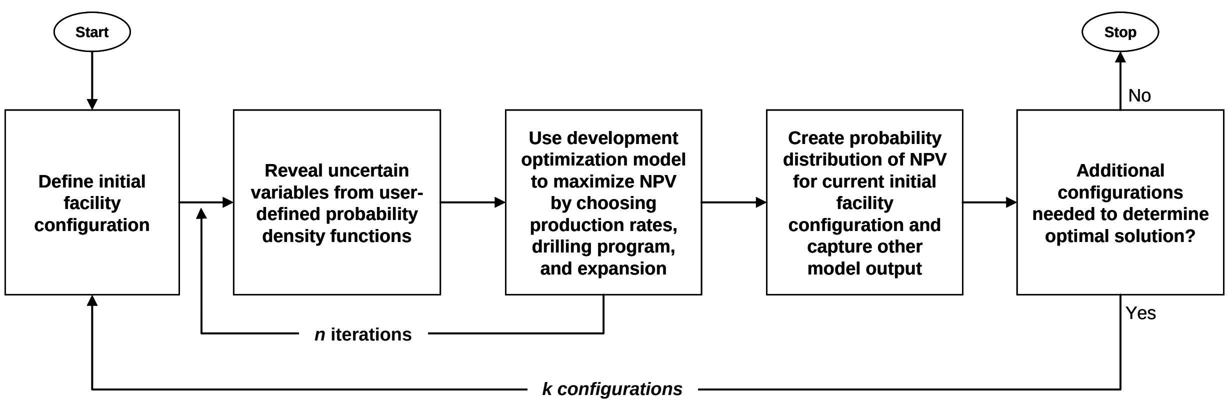

Figure 36: Flow chart of uncertainty analysis procedure

Figure 25 shows the uncertainty analysis workflow procedure using the model presented in this thesis.

Case Studies

As stated in the introduction to this chapter, the objective is to investigate the impact of uncertainty in the reservoir properties on decisions concerning the choice of the initial facility size, the number of rigs and the optimum predrilled wells. For this study the reservoir properties of interest are the net to gross ratio and the degree of communication between the good and bad portion of the reservoir. We restrict the uncertainty analysis to Model 2.

The three questions we aim to answer in this section are:

1. What is the optimal initial facility capacity given the uncertainty in the net to gross ratio and the degree of communication between the good and bad portions of the reservoir?
2. Given an optimal initial facility capacity, what is the optimal number of predrilled wells given the uncertainty in the net to gross ratio and the degree of communication between the good and bad portions of the reservoir?
3. Given an optimal initial facility capacity and optimal number of predrilled wells, what is the optimal number of rigs given the uncertainty in the net to gross ratio and the degree of communication between the good and bad portions of the reservoir?

Model 2

Input reservoir and economic data: Per the definition of Model 2, the reservoir is split into two tanks of varying quality. Compartment 1 represents the good portion of the reservoir and compartment 2 represents the bad portion. The compartments differ in their porosity, permeability, and initial oil saturation.

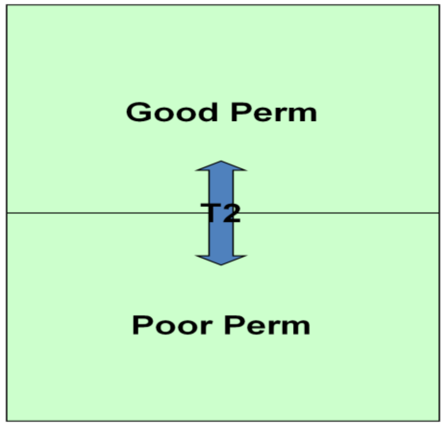

Figure 37: Simplified representation of reservoir to capture heterogeneity

The net thickness in both compartments is defined as a uniformly distributed random variables between 200 and 400 ft and the transmissibility between the two compartments is also uniformly distributed between 0 and 1000 bbl/d/psi. Table 16 presents the reservoir properties in each compartment.

Table 16: Reservoir input properties for uncertainty analysis

Single Phase Flow		
Property	**Compartment 1**	**Compartment 2**
Initial pressure, P_i (psi)	20000	20000
Porosity, (fraction)	0.19	0.15
Wellbore radius, r_w (ft)	0.328	0.328
C_A	30.1	30.1
Skin, s (dimensionless)	-0.91	-0.91
Permeability, k (md)	8	4
Viscosity, (cp)	1.7	1.7
Total compressibility, c_t (psi^{-1})	2.25 x10^{-5}	2.25 x10^{-5}
Initial oil saturation, S_{oi}	0.79	0.69

The initial facility size is defined to take on seven different values which are presented in table 17.

Table 17: Range of values for initial facility size

Initial Facility Size (bbl/d)	25000	50000	100000	150000	200000	250000	300000

It should be noted that for this analysis the expansion of the facility is allowed in year nine (four years after initial production). No more than three wells can be drilled in a year (one rig constraint). The facility cost occurred in year three and there is no production until year six, it was assumed that it takes five years to construct and install the production facility and that drilling and production starts at year 5. The economic parameters used for this analysis are presented in table 18.

Table 18: Cost function parameters for initial facility sizing analysis

Cost parameter	**Value**
Discount rate, r (%)	7.5%
Well cost (US Dollars)	$250x10^6$
Facility construction coefficient one	$150x10^6$
Facility construction coefficient two	$14x10^6$
Facility expansion coefficient one	$150x10^6$
Facility expansion coefficient two	$14x10^6$
Variable production cost (US Dollars)	8.0
Expansion cost multiplier	0.5
Expansion capacity multiplier	2.0

Initial facility capacity analysis

Base case: In the base case the expected value of the distribution of the net to gross ratio and the transmissibility are used along with the workflow to determine the optimal initial facility capacity.

NTG case: For this case all the uncertain variables are fixed at their expected value and only the net to gross ratio is defined as a stochastic variable. The purpose of this case is to investigate the impact of uncertainty in the NTG ratio on the choice of the initial facility capacity.

TT case: For this case all the uncertain variables are fixed at their expected value and only the transmissibility is defined as a stochastic variable. The purpose of this case is to investigate the impact of uncertainty in the transmissibility between the good and bad compartment of the reservoir on the choice of the initial facility capacity.

NTGTT case: For this case the net to gross ratio and transmissibility are defined as a stochastic variables. The purpose of this case is to investigate the simultaneous impact of uncertainty in the NTG ratio and transmissibility on the choice of the initial facility capacity.

The results obtained for these cases are presented in the figure below.

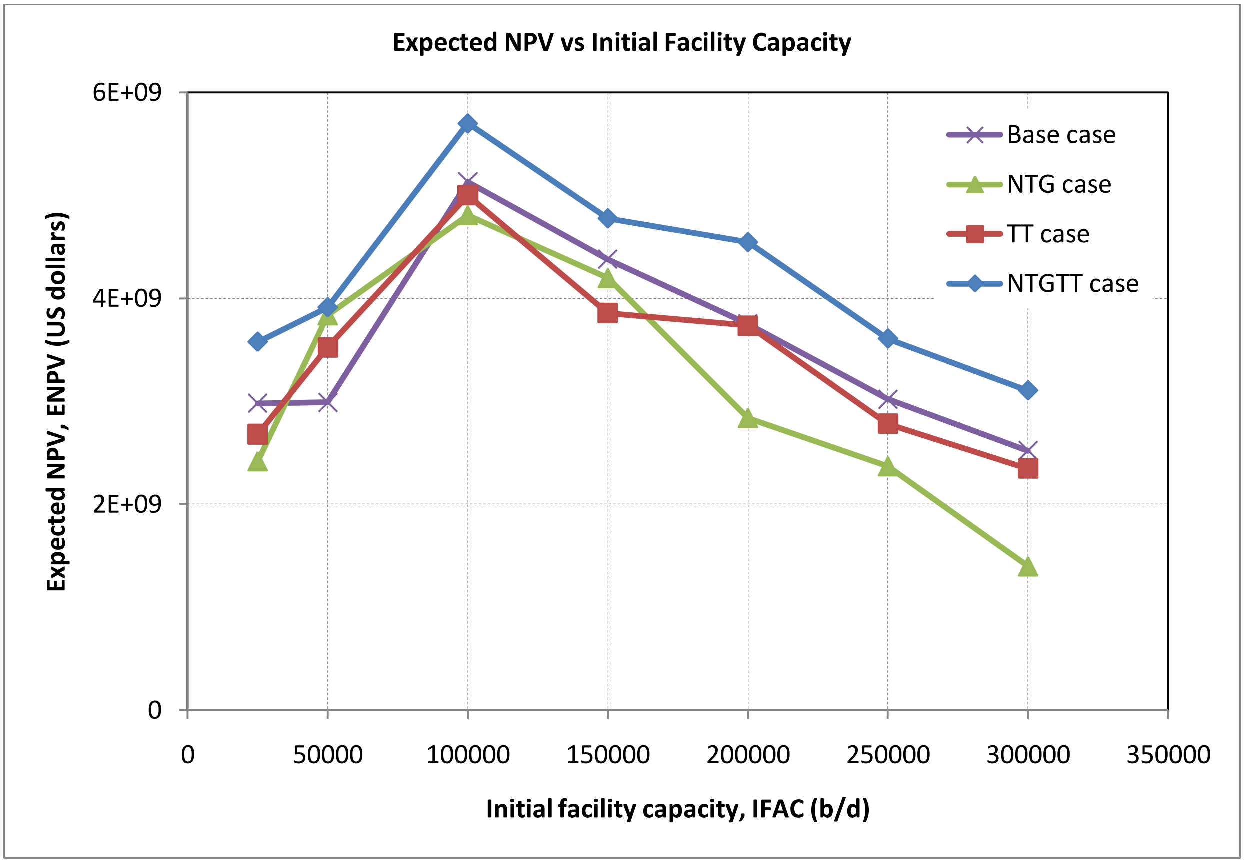

Figure 38: Expected NPV vs initial facility capacity, optimal initial facility analysis

From figure 38, it is seen that as the initial facility capacity increases the expected NPV increases until the initial facility capacity is equal to 100000 bbl/d after which the expected NPV starts to decrease. The expected NPV is slightly lower in both the NTG and TT cases when compared to the base case while it is higher in the NTGTT case. In all the cases considered, the expected NPV attains it a maximum value when the initial facility capacity is 100000 bbl/d. Thus, the optimal initial facility capacity is 100000 bbl/d. Standard deviation in the NPV is shown in table 18 below and the cumulative oil recovered is shown in figure 39.

Table 19: Standard deviation of NPV, initial facility capacity analysis

IFAC (bbl/d)	NTG case	TT case	NTGTT case
25000	1.50×10^9	1.59×10^9	7.82×10^8
50000	1.82×10^9	1.44×10^9	1.58×10^9
100000	1.33×10^9	1.46×10^8	1.15×10^9
150000	1.39×10^9	8.33×10^8	1.47×10^9
200000	2.18×10^9	1.62×10^8	1.25×10^9
250000	1.84×10^9	3.97×10^8	1.31×10^9
300000	1.91×10^9	1.47×10^8	1.21×10^9

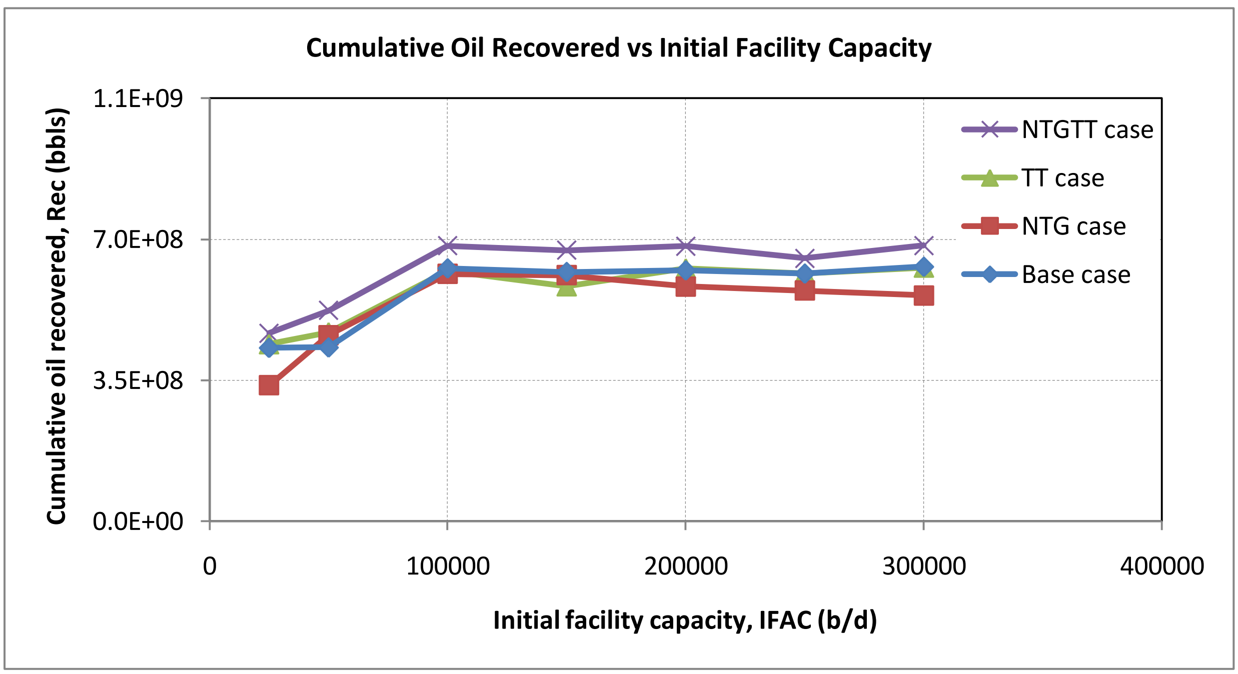

Figure 39: Cumulative oil recovered versus initial facility capacity, initial facility capacity analysis

In all the cases considered the cumulative oil recovered increases with the initial facility capacity until the initial facility capacity is equal to 100000 bbl/d and the cumulative oil recovered stays relatively constant. This result suggests that there is a threshold value of the initial facility capacity above which the cumulative oil recovered is approximately constant. For all the cases considered the threshold value of initial facility capacity is 100000 bbl/d. A comparison of the total number of well drilled for each case is shown in figure 40. The base, NTG and TT case drilled a maximum of 19 wells when the initial facility capacity was 100000 bbl/d, 100000 bbl/d and 150000 bbl/d respectively while the NTGTT case drilled a maximum of 21 wells when the initial facility capacity was 100000 bbl/d.

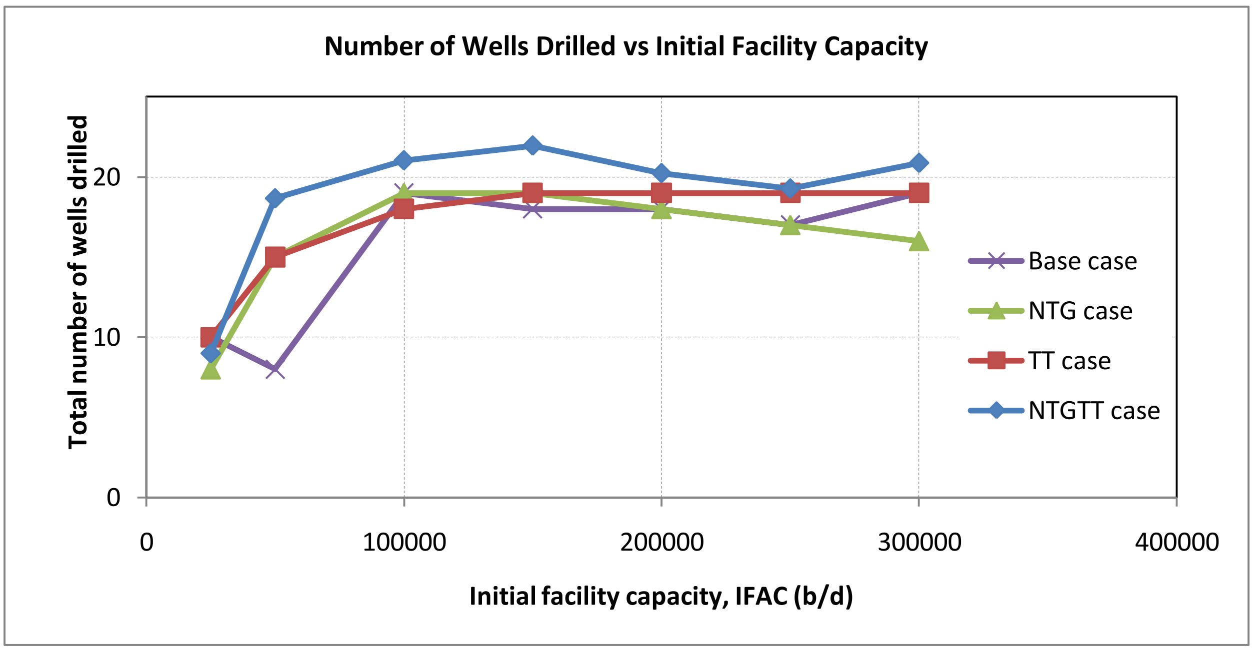

Figure 40: Total number of wells drilled versus initial facility capacity

Table 19 summarizes the probability of expansion given the choice of initial facility capacity. From this table it is obvious that the larger the initial facility capacity, the lower the probability of expansion. And the smaller the initial facility capacity, the more likely the facility capacity will be expanded.

Table 20: Probability of expansion given the choice of initial facility capacity

IFAC (bbl/d)	***NTG***	***TT***	***NTGTT***
25000	0.64	0.90	0.96
50000	0.76	0.63	0.71
100000	0.64	0.88	0.84
150000	0.04	0.00	0.08
200000	0.00	0.00	0.00
250000	0.00	0.00	0.00
300000	0.00	0.00	0.00

Pre - drill well analysis

Base case: In the base case the expected value of the distribution of the net to gross ratio and the transmissibility are used along with the workflow to determine the optimal number of wells to be pre-drilled. The facility capacity is set equal to the optimal value (100000 bbl/d) as determined from the analysis above.

NTG case: For this case all the uncertain variables are fixed at their expected value and only the net to gross ratio is defined as a stochastic variable. The purpose of this case is to investigate the impact of uncertainty in the NTG ratio on the optimal number of pre-drilled wells.

TT case: For this case all the uncertain variables are fixed at their expected value and only the transmissibility is defined as a stochastic variable. The purpose of this case is to investigate the impact of uncertainty in the transmissibility between the good and bad compartments of the reservoir on the optimal number of pre-drilled wells.

NTGTT case: For this case the net to gross ratio and transmissibility are defined as a stochastic variables. The purpose of this case is to investigate the impact of uncertainty in the NTG ratio and transmissibility on the optimal number of pre-drilled wells.

The results for this analysis are presented below;

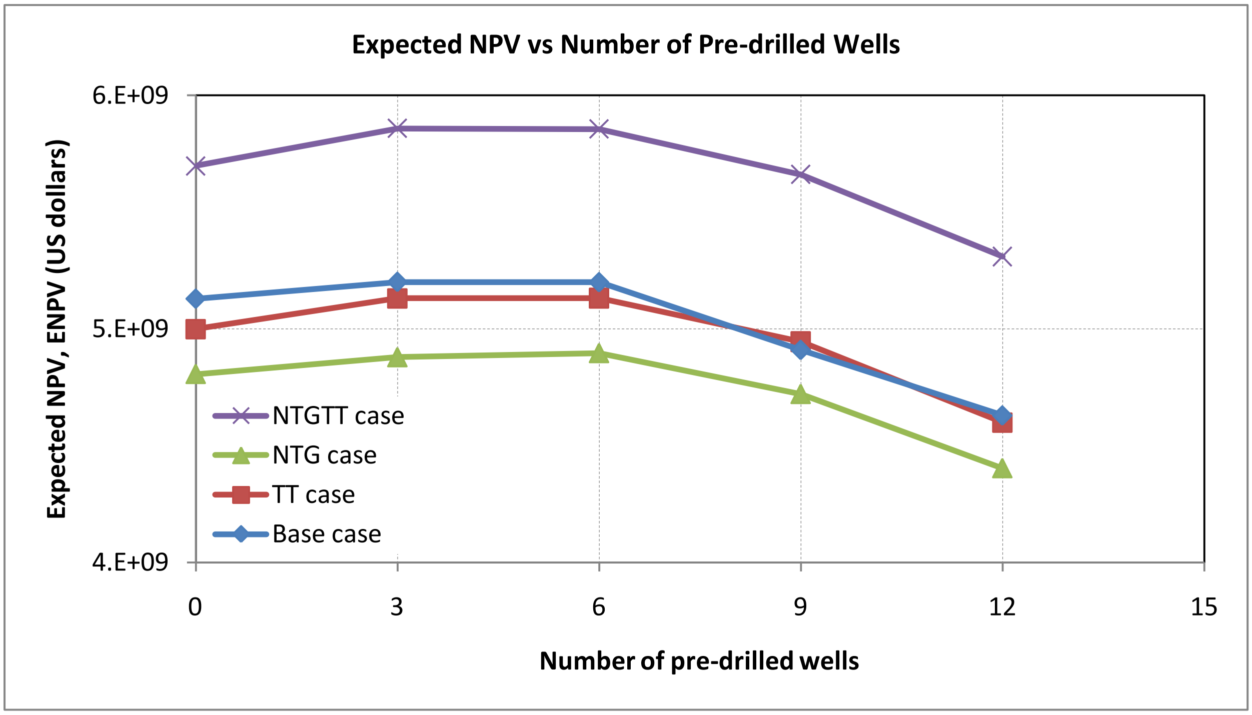

Figure 41: Expected NPV versus number of pre-drilled wells, optimal pre-drilled well analysis

In all the case, the expected NPV gradually increases as the number of pre-drilled wells increased until the number of predrilled wells is six. When the number of predrilled wells is greater than six the expected NPV begins to decrease. This is because in the net present value formula, cash flows that occur early have a significant impact on the value of the NPV. Based on the expected NPV, the optimal number of predrilled wells in both the NTG and TT case is six (their respective NPVs are USD 4.9 $x10^9$ and USD 5.1 $x10^9$) while it is three for the base and

NTGTT (their respective NPVs are USD 5.9 x10^9 and USD 5.2 x10^9). The standard deviation of the NPV (in USD) is presented in table 20.

Table 21: Standard deviation of NPV, optimal pre-drilled wells analysis

IFAC (bbl/d)	***NTG case***	***TT case***	***NTGTT case***
0	1.33 x10^9	1.46 x10^8	1.15 x10^9
3	1.31 x10^9	1.37 x10^8	1.16 x10^9
6	1.30 x10^9	9.96 x10^7	1.08 x10^9
9	1.25 x10^9	7.58 x10^7	1.04 x10^9
12	1.22 x10^9	4.44 x10^7	9.96 x10^8

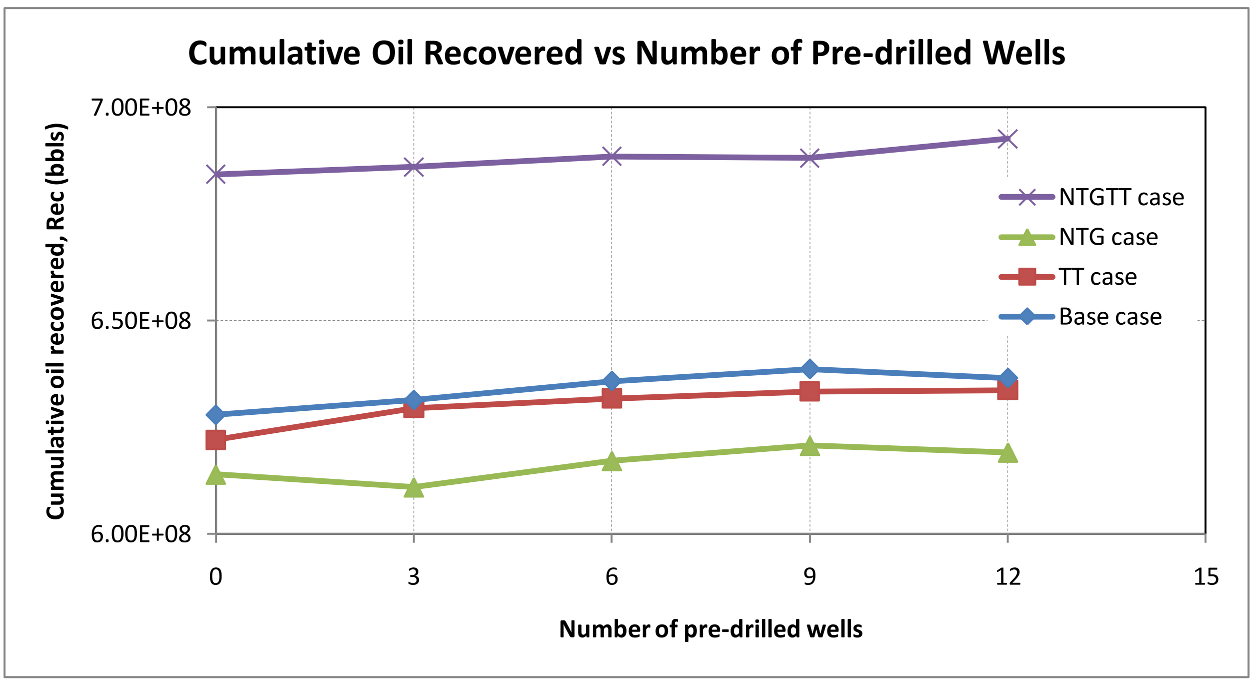

Figure 42: Cumulative oil recovered versus number of pre-drilled wells, optimal pre-drilled well analysis

Generally, in all the cases considered the cumulative oil recovered increased gradually as the number of predrilled wells increased, figure 42. For the base and NTG cases the cumulative oil recovered was a maximum when the number of predrilled wells was nine while it was a maximum at 12 predrilled wells for the TT and NTGTT cases. A comparison of the total number of additional wells drilled for each case is shown in figure 43. As the number of predrilled wells increased the number of additional wells drilled decreased.

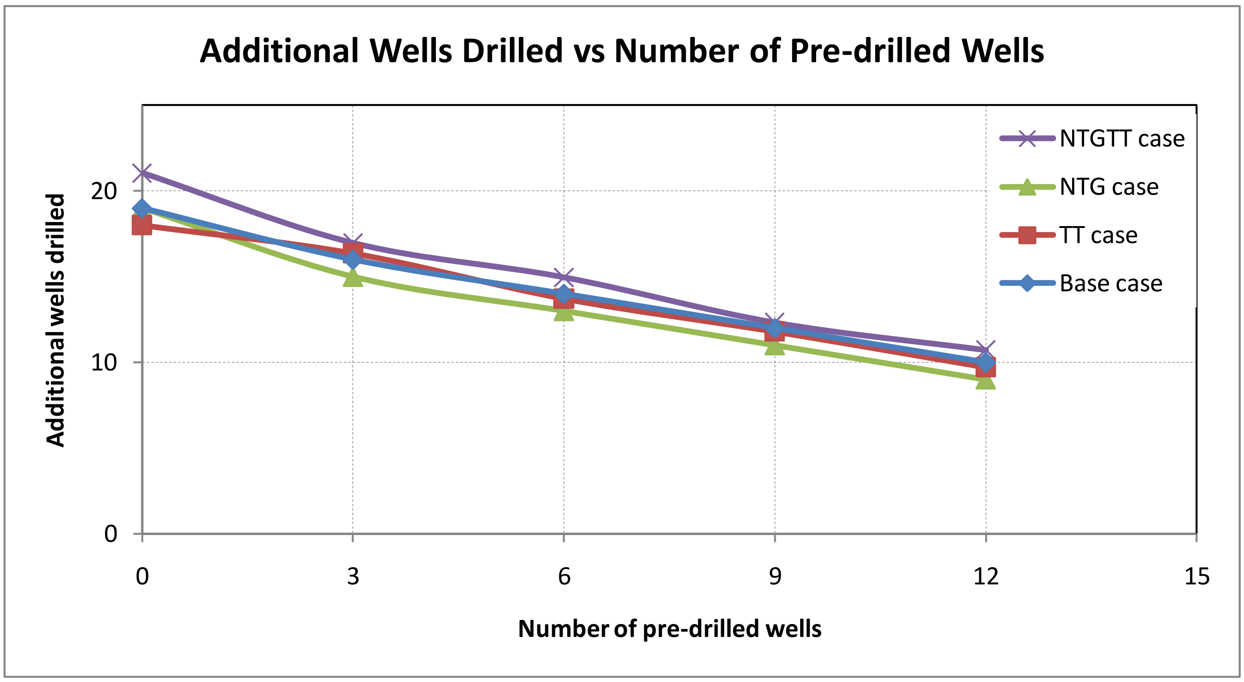

Figure 43: Total number of wells drilled versus number of pre-drilled wells, optimal pre-drilled well analysis

Rig count analysis

Base case: In the base case the uncertain variables (net to gross ratio and transmissibility) are fixed at their expected value. The facility capacity is set equal to the optimal value 100000 bbl/d and three wells are pre-drilled as determined from the analysis above.

NTG case: For this case all the uncertain variables are fixed at their expected value and only the net to gross ratio is defined as a stochastic variable. The purpose of this case is to investigate the impact of uncertainty in the NTG ratio on the optimal number of number rigs.

TT case: This case is identical to the base case and only the transmissibility is defined as a stochastic variable. The purpose of this case is to investigate the impact of uncertainty in the transmissibility between the good and bad compartments of the reservoir on the optimal number of rigs.

NTGTT case: This case is identical to the base case except that the net to gross ratio and transmissibility are defined as stochastic variables. The purpose of this case is to investigate the impact of uncertainty in the NTG ratio and transmissibility on the optimal number of rigs.

The result for this analysis is presented below;

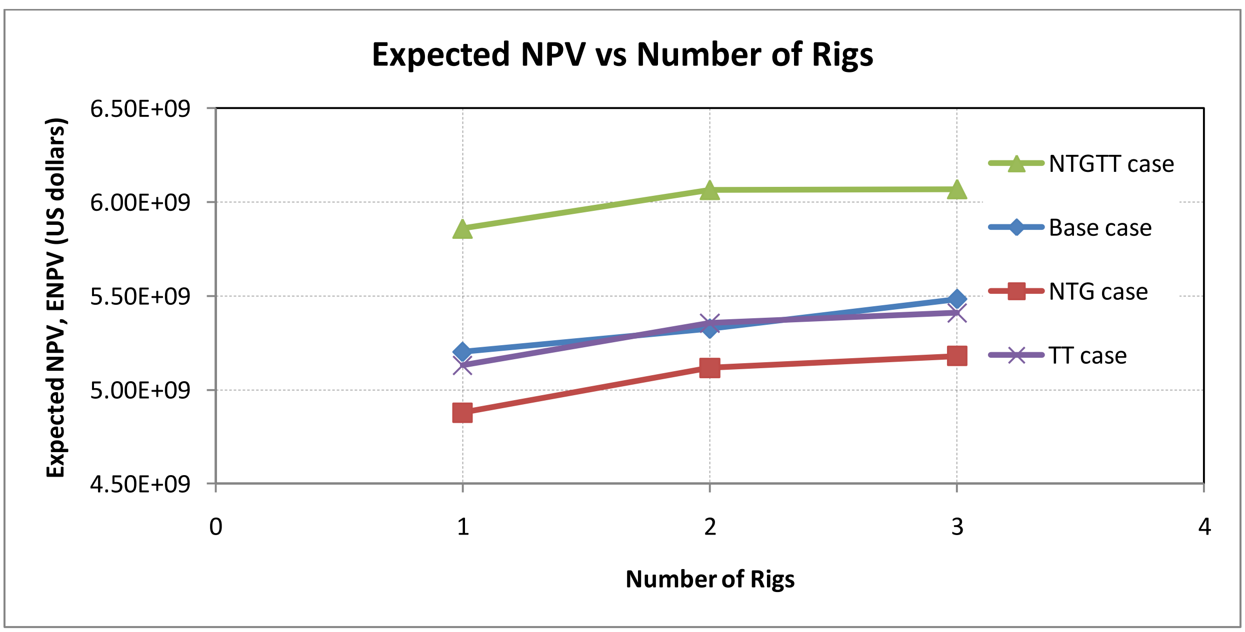

Figure 44: Expected NPV versus number of rigs, optimal rig count analysis

In the NTG, TT and Base cases, the expected NPV increases slightly as the number of rigs increased from one to three while in the NTGTT case, the NPV increased when the number of rigs increased from one to two and stayed relatively constant when the number of rigs was three. The optimal number of rigs is 3 for the base, NTG and TT cases. Individually the NTG and TT have little or no impact on the choice of the number of rigs. On the other hand when both NTG and TT are uncertain the optimal number of rigs is 2. The standard deviation of the NPV (in USD) is presented in table 21.

Table 22: Standard deviation of NPV, optimal rig count analysis

Number of Rigs	*NTG case*	*TT case*	*NTGTT case*
1	1.31×10^9	1.37×10^8	1.16×10^9
2	1.29×10^9	1.14×10^8	1.10×10^9
3	1.26×10^9	1.17×10^8	1.44×10^9

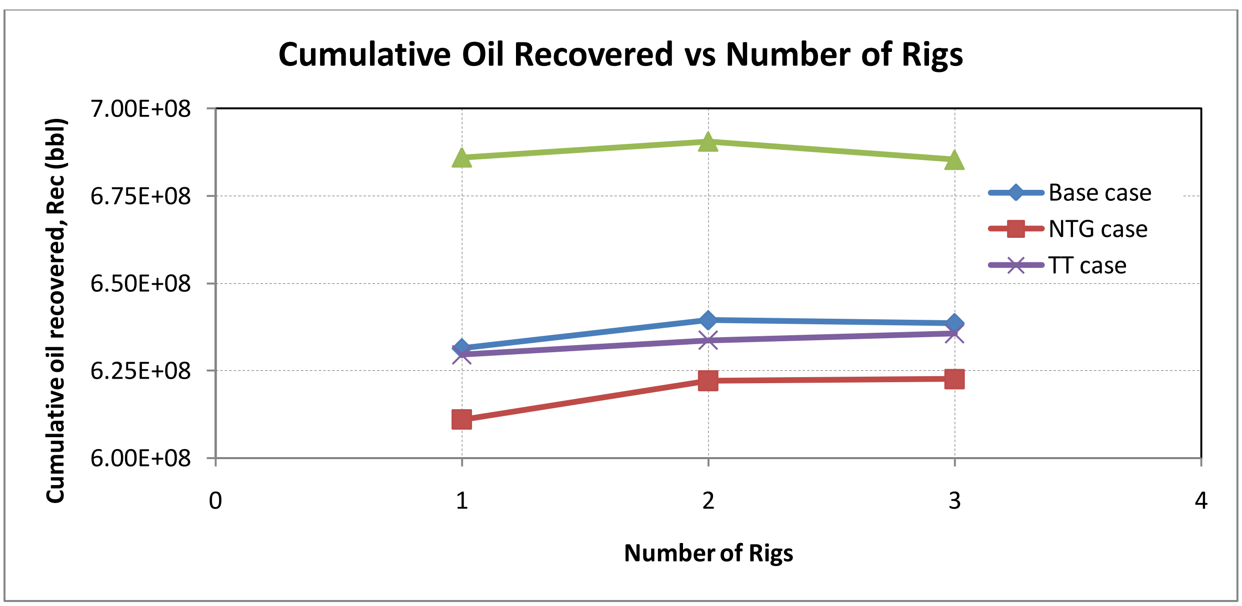

Figure 45: Cumulative oil recovered versus number of rigs, optimal rig count analysis

In the base case the cumulative oil recovered increased slightly as the number of rigs increased from one to two and stayed constant as the number of rigs increased from two to three. A similar profile is observed in the NTG case. The cumulative oil recovered in the TT case increased slightly as the number of rigs increased from one to three. The oil recovered in the NTGTT case increased as the number of rigs increased from one to two and decrease as the number of rigs increased from two to three. If the cumulative oil recovered was the objective function, the optimal number of rigs in the base, NTG, TT and NTGTT case respectively would be 2, 2, 3 and 2. There is thus a trade off in the optimal decision if the objective function is changed from the NPV to the cumulative oil recovered. In descending order the cumulative oil recovered is highest in the NTGTT case followed by the base case and the TT case. The NTG case recovered the smallest amount of oil, figure 45. A comparison of the total number of additional wells drilled for each case is shown in figure 46. In all the cases the maximum number of wells is drilled when the number of rigs is 2.

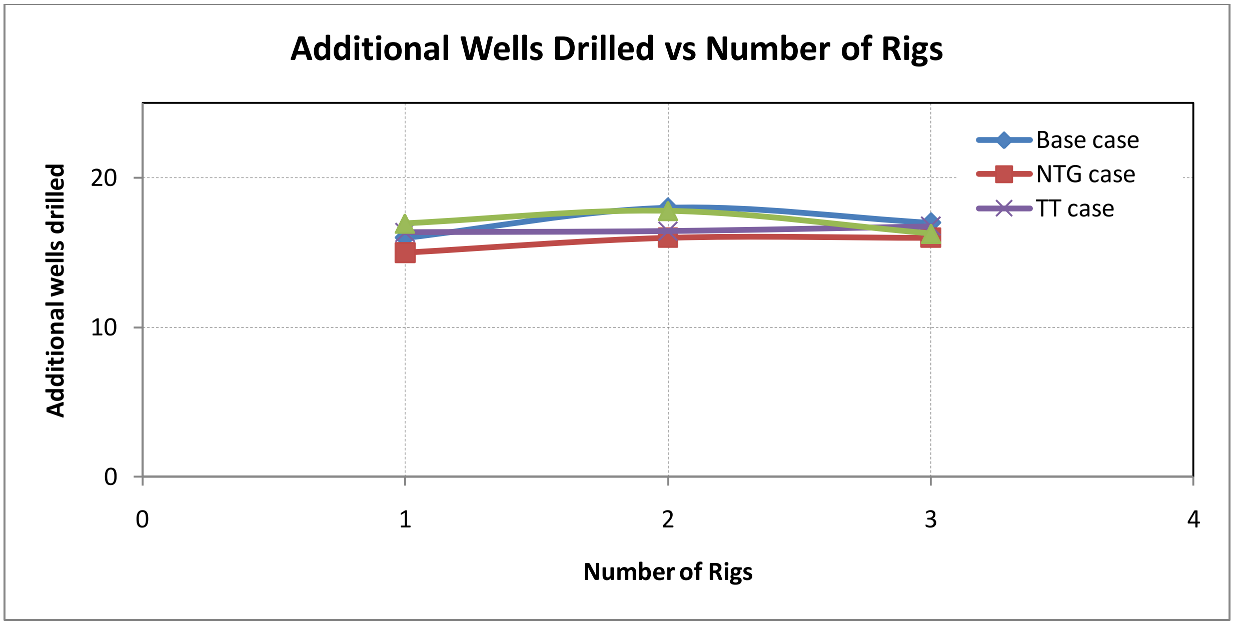

Figure 46: Additional number of wells drilled versus number of rigs, optimal rig count analysis

CHAPTER SIX: CONCLUSION AND FUTURE WORK

Conclusion

1. A simple tool was developed to help make decisions at the concept selection phase of a project. The tool is an integrated asset model (IAM) developed by coupling a simple reservoir tank model to a facility and economic model. The IAM was formulated as an optimization problem to maximize the net present value subject to reservoir, facilities and economic constraints.
2. The IAM was used to study the impact of uncertainty in reservoir properties (net to gross ratio and transmissibility) on decisions concerning the initial facility capacity, the number of wells to pre-drill and the number of rigs to contract.
3. Uncertainty in reservoir properties can impact decision concerning the optimal facility capacity, the optimal number of predrilled wells and the optimal number of rigs.
4. The model can also help identify trade-offs between decisions for different performance metrics.
5. The model output provides a range of optimal values for the decision under consideration (initial facility capacity, number of pre-drilled wells and rig count) and it is easy to analyze. The results can also help identify areas for detailed study before decisions are made.

Recommendations for future work

1. A theoretical background for n-compartments was presented in this work and the current model has been shown to work well for 2 compartments. In the future the model can be demonstrated for more than 2 tanks.

2. This study was restricted to the single phase flow problem because there were difficulties encountered when the two phase flow problem was considered. The two phase flow problem was found to work for certain conditions and it broke down when these conditions are altered. Also, when the two phase model does solve, water production is observed instantaneously at the producing wells because of syntax restriction imposed by GAMS. The syntax restriction prevents the implementation of the breakthrough condition on the model equation. A work around was implemented by multiplying the water production rate equation by a step function that is zero when the average water saturation in the reservoir is less than the average water saturation at break through as given by the Buckley Leveret solution and equal to one when the relationship is equal to. Developing a work around for the instantaneous water production and understanding why the model breaks down when two phase flow is involved could be an exciting area for future research.
3. The current 2 tank model solves a 25 iteration problem in about 5 hours, this is computationally expensive. Implementing a decomposition algorithm such as the Benders decomposition might reduce the solution time. Hence it will be a worthy investment of time to reformulate the problem and apply a different solution algorithm to speed it up.
4. Some might argue that the model is too simple to fully account for reservoir behavior but several authors have also argued that given the dearth of information about the reservoir at the concept selection phase of a project, these simplified models are adequate representations of the reservoir. An interesting study will be one showing the equivalence of these simple models to the more sophisticated models. Such a study could highlight

when the simple tank models are as good as, better, or not as good as the complex reservoir simulators and when it is appropriate to use either one.

5. A future study could apply a proxy model/response surface in place of the tank model.
6. It may also be worthwhile to use the IAM as the core of a conventional experimental design- response surface workflow. That is, the number of iterations could be reduced via an experimental design, thus reducing the computational cost.

APPENDIX A: INTEGRATED ASSET MODEL CODE IN GAMS

```
$Title A Simple Optimization Model for a Solution Gas Drive Reservoir
$Onsymlist onsymxref onuellist onuelxref
$eolcom !
set
        t       Time                            /1*35/  !Defines expected field life
        n       Number of trials used with random variable      /1/
        nw      Index for wells                 /1*30/  !Set is used to index total wells in the field
        t1(t)
        ;
        t1(t)=yes$(ord(t)eq 1);

alias (t,ta);

********************************************************************************
Scalars
*----------------------------------------------------------------------------
*Reservoir constants
*----------------------------------------------------------------------------
*Compartment 1
*$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
        Pinit1  Reservoir pressure at time t = 0 psia   /20000/   !Initial reservoir pressure
        Phi1    Porosity                        /0.19/
        rw1     Well radius ft                  /0.328/
        Ca1     Dietz shape factor              /30.1/
```

```
s1          Skin factor                         /-0.91/
k1          Absolute permeability mD                      /8/
uo1          Oil viscosity                        /1.7/
Ct1          Total compressibility per psi              /2.25E-5/
Soi1                                        /0.79/
*----------------------------------------------------------------------------
*Compartment 2
*$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$$$$
Pinit2        Reservoir pressure at time t = 0 psia         /20000/    !Initial reservoir pressure
Phi2          Porosity                            /0.15/
rw2           Well radius ft                         /0.328/
Ca2           Dietz shape factor                       /30.1/
s2          Skin factor                         /-0.91/
k2          Absolute permeability mD                      /4/
uo2          Oil viscosity                        /1.7/
Ct2          Total compressibility per psi              /2.25E-5/
Soi2                                        /0.69/
*----------------------------------------------------------------------------
*Cost function constants
*----------------------------------------------------------------------------
Discountrate              Discount rate                                        /0.075/
Fixedwellcost             Fixed cost of well (million USD)                              /250E6/
Facilityconstructioncoef1           Facility construction cost parameter one (million USD)
/150E6/
Facilityconstructioncoef2           Facility construction cost parameter two (million USD)
/14E6/
Facilityexpansioncoef1                Facility expansion cost parameter one (million USD)
/150E6/
```

Facilityexpansioncoef2 Facility expansion cost parameter two (million USD) varies with facility size /14E6/

Variableproductioncost Variable production cost USD per bbl /8/

Expansioncostmultiplier Expansion cost multiplier /0.5/

Expansioncapacitymultiplier Expansion capacity multiplier /2.0/

maxrate1

maxrate2

;

Parameters

*---

*Reservoir parameters

*---

*Compartment 1

*$$$

Vp1 Reservoir pore volume bbls

ResArea1 Reservoir area acres

Area1 Well drainage area acres

j1(nw) Well productivity index bbls per day per psi

OOIP1 Oil originally in-place bbls

So1(t) Oil saturation

h1 Average perforation thickness ft

b1 Average breath of compartment one ft

Fperf Fraction of net thickness perforated

*---

*Compartment 1

```
*$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$$$$
        Vp2             Reservoir pore volume bbls
        ResArea2        Reservoir area acres
        Area2           Well drainage area acres
        j2(nw)          Well productivity index bbls per day per psi
        OOIP2           Oil originally in-place bbls
        So2(t)          Oil saturation
        h2              Average perforation thickness ft
        b2              Average breath of compartment two ft
        Dt
        T12
*------------------------------------------------------------------------------
*Cost function parameters
*------------------------------------------------------------------------------
        Price(t)        Oil price in dollars per bbl varied with time
        Discountfactor(t)  Discount factor
        Initialfacilitycap Initial facility capacity 1000 bopy
        Maximumwellslot    Maximum possible slots on platform
        Expansiontime      Expansion cannot be done before this time period (this value cannot be
1)
        Platformcost(t)
        Twells             Total number of wells that can be drilled in the field
        Pwf                Bottomhole flowing pressure
;
*------------------------------------------------------------------------------
*------------------------------------------------------------------------------
        Discountfactor(t) = 1/((1+Discountrate)**ord(t));      !This calculates the discount factor
        Dt   = 365;
```

```
        Fperf = 1;
        Twells = 35;
        Pwf = 12500;
Variables
*------------------------------------------------------------------------------
*Reservoir variables
*------------------------------------------------------------------------------
        z                    Net present value
        Capacityaddswitch(t)    1 if expansion is required 0 otherwise
        Nxf(t)
        Qxf(t)
        zs
        TT
Positive variables
        P1(t)                 Reservoir pressure psi
        Np1(t)                  Total cumulative production from the field in stb
        Q1(t,nw)                  Individual well production rate stb per day
        QT(t)
        Pwf1(t,nw)                Bottomhole flowing pressure for individual wells psia
        P2(t)                 Reservoir pressure psi
        Np2(t)                  Total cumulative production from the field in stb
        Q2(t,nw)                  Individual well production rate stb per day
        Pwf2(t,nw)                Bottomhole flowing pressure for individual wells psia
        Np(t)
*------------------------------------------------------------------------------
*Cost function variables
*------------------------------------------------------------------------------
        Facilitycapacity(t)    Facility capacity (1000 bopy)
        Extracapacity(t)       Expansion capacity (1000 bopy)
```

```
        TotalCAPEX(t)          Total capital cost (million USD)
        TotalOPEX(t)          Tdotal operating cost (1000 USD)
        Wellcost(t)          Drilling cost (million USD)
        Expansioncost(t)       Expansion cost (million USD)
        wellcap1(t,nw)         Controls the capacity by each well
        wellcap2(t,nw)         Controls the capacity by each well
        Tcost(t)
;
*============================================================================
==
Binary variable
        Capacityaddswitch(t), dr1(t,nw),dr2(t,nw);
Equations
*---------------------------------------------------------------------------
*Reservoir model
*---------------------------------------------------------------------------
        NPV                    This is the objective function and it models the net present value NPV
        Pressure1(t,nw)              Describes reservoir pressure profile
        Pressure2(t,nw)

        qwell1(t,nw)                Models individual well production rate
        qwell2(t,nw)
        qxflow(t)

        cumulativeproduction1(t)
        cumulativeproduction2(t)
        cumulativecrossflow(t)
        cumulativeproduction(t)
*---------------------------------------------------------------------------
```

CAPEXmodel(t) Models the capital expenditure

Wellcostmodel(t) Defines the cost of drilling wells

OPEXmodel(t) Models the operating expenditure

*--

Facilityexpansioncost(t) Models the cost of facility expansion

FACexpansionconstraintone(t) First constraint used to implement facility expansion in expansion year

FACexpansionconstrainttwo(t) Second constraint used to implement facility expansion in expansion year

FACexpansionconstraintthree(t) Third constraint used to implement facility expansion in expansion year

FACexpansionconstraintfour(t) Fourth constraint used to implement facility expansion in expansion year

FACexpansionconstraintfive(t) Fifth constraint used to implement facility expansion in expansion year

FACexpansionconstraintsix(t) Sixth constraint used to implement facility expansion in expansion year

Drillingwellconstraint2(t)

onewell1(nw)

onewell2(nw)

DrilleqnA1(t,nw)

DrilleqnA2(t,nw)

Totalprod(t)

DrilleqnB

BHP1(t,nw)

BHP2(t,nw)

Totalcost(t)

;

*--

```
NPV..                                        z  =e=  sum(t,((price(t)*(Np(t)-Np(t-1))-TotalOPEX(t))  -
TotalCAPEX(t))*Discountfactor(t)) - Platformcost('3');

qwell1(t,nw)..          Q1(t,nw) =e= j1(nw) * (P1(t)$(not t1(t)) + Pinit1$t1(t) - Pwf1(t,nw));
qwell2(t,nw)..          Q2(t,nw) =e= j2(nw) * (P2(t)$(not t1(t)) + Pinit2$t1(t) - Pwf2(t,nw));
qxflow(t)..            Qxf(t) =e= T12*(P1(t)-P2(t));

cumulativeproduction1(t)..   Np1(t) =e= Np1(t-1) + (sum((nw), Q1(t,nw))*365);
cumulativeproduction2(t)..   Np2(t) =e= Np2(t-1) + (sum((nw), Q2(t,nw))*365);
cumulativecrossflow(t)..     Nxf(t) =e= Nxf(t-1) + (Qxf(t)*365);
cumulativeproduction(t)..    Np(t)  =e= Np1(t) + Np2(t);

Pressure1(t,nw)..         P1(t)   =e=  Pinit1 - (Np1(t) + Nxf(t))/(Vp1*Ct1);
Pressure2(t,nw)..         P2(t)   =e=  Pinit2 - (Np2(t) - Nxf(t))/(Vp2*Ct2);

Totalprod(t)..           QT(t) =e= sum(nw, Q1(t,nw)) + sum(nw, Q2(t,nw));

BHP1(t,nw)..             Pwf1(t,nw) =g= Pwf;
BHP2(t,nw)..             Pwf2(t,nw) =g= Pwf;

onewell1(nw)..            sum(t,dr1(t,nw)) =l= 1;
onewell2(nw)..            sum(t,dr2(t,nw)) =l= 1;
* This next equation is the annual drilling constraint:
Drillingwellconstraint2(t)..  sum(nw,dr2(t,nw))+ sum(nw,dr1(t,nw))=l= 6;

*This next set of equations makes sure we attach costs to wells in the periods
*when the drilling is done
DrilleqnA1(t,nw)..        Q1(t,nw)=l= maxrate1*sum(ta$(ord(ta)<=ord(t)), dr1(ta,nw));
DrilleqnA2(t,nw)..        Q2(t,nw)=l= maxrate2*sum(ta$(ord(ta)<=ord(t)), dr2(ta,nw));
```

```
DrilleqnB..          sum((nw,t),dr2(t,nw))+ sum((nw,t),dr1(t,nw))=l= Twells;
*-------------------------------------------------------------------------------
CAPEXmodel(t)..          TotalCAPEX(t)    =e= wellcost(t) + Expansioncost(t);
Wellcostmodel(t)..          Wellcost(t)      =e= Fixedwellcost * (sum(nw, dr1(t,nw)) + sum(nw,
dr2(t,nw)));
OPEXmodel(t)..          TotalOPEX(t)    =e= variableproductioncost * (Np(t)-Np(t-1));
*-------------------------------------------------------------------------------
FACexpansionconstraintone(t)..                    QT(t) =l= Facilitycapacity(t);
FACexpansionconstraintfour(t)$(ord(t)  gt  Expansiontime)..          Facilitycapacity(t)  =e=
Facilitycapacity(t-1);
FACexpansionconstrainttwo(t)$(ord(t)  eq  Expansiontime)..          Facilitycapacity(t)  =e=
Facilitycapacity(t-1) + Extracapacity(t);
FACexpansionconstraintthree(t)$(ord(t)  lt  Expansiontime)..          Facilitycapacity(t)  =e=
Initialfacilitycap;
FACexpansionconstraintfive(t)..                              Extracapacity(t)      =l=
Expansioncapacitymultiplier*Initialfacilitycap*
                                                  Capacityaddswitch(t);
FACexpansionconstraintsix(t)..                              Facilitycapacity(t)  =l=
Expansioncapacitymultiplier*Initialfacilitycap;
Facilityexpansioncost(t)..                         Expansioncost(t)            =e=
Expansioncostmultiplier*((Capacityaddswitch(t)*Facilityexpansioncoef1)+(Extracapacity(t)/100
0*
                              Facilityexpansioncoef2));
Totalcost(t)..     Tcost(t) =e= TotalCAPEX(t) + TotalOPEX(t) + Platformcost(t);


Model Reservoirmodel /all/;
*option MINLP = BARON;
Loop(n,
```

```
Price(t)            = 50;

h1 = uniform(200,400);
h2 = uniform(200,400);

T12 = uniform(0,1000);

QT.fx('1') = 0;
QT.fx('2') = 0;
QT.fx('3') = 0;
QT.fx('4') = 0;
QT.fx('5') = 0;
dr1.fx('1',nw)=0;
dr1.fx('2',nw)=0;
dr1.fx('3',nw)=0;
dr1.fx('4',nw)=0;
dr1.fx('5',nw)=0;
dr2.fx('1',nw)=0;
dr2.fx('2',nw)=0;
dr2.fx('3',nw)=0;
dr2.fx('4',nw)=0;
dr2.fx('5',nw)=0;

dr1.fx('2','2')=1;
dr1.fx('3','3')=1;
dr2.fx('3','3')=1;

dr1.fx('2','4')=1;
dr1.fx('3','5')=1;
```

```
dr2.fx('4','6')=1;

dr1.fx('2','8')=1;
dr2.fx('4','8')=1;
dr2.fx('4','7')=1;

dr1.fx('1','10')=1;
dr2.fx('1','10')=1;
dr2.fx('1','12')=1;

ResArea1 = 5000;          !Area of compartment 1 in arces
ResArea2 = 5000;          !Area of compartment 2 in arces

Vp1    = 7758*ResArea1*h1*phi1;      !This randomly picks a value for the reservoir pore
volume
Vp2   = 7758*ResArea2*h2*phi2;

Area1 = ResArea1/Card(nw);
Area2 = ResArea2/Card(nw);

OOIP1 = Vp1*Soi1;
OOIP2 = Vp2*Soi2;

maxrate1    =(0.00708*k1*Fperf*h1)/(uo1*(0.5*log((Area1)/(rw1**2*Ca1))+5.75+s1))    *
(Pinit1);
maxrate2    =(0.00708*k2*Fperf*h2)/(uo2*(0.5*log((Area2)/(rw2**2*Ca2))+5.75+s2))    *
(Pinit2);

Expansiontime       = 9;
```

```
Capacityaddswitch.l(t)= 0;

Initialfacilitycap    = 100000;
maximumwellslot       = 0;
Platformcost('3')= (Facilityconstructioncoef1 + (Facilityconstructioncoef2 * Initialfacilitycap/1000) + (22.5E6*maximumwellslot));

dr1.l(t,nw) = round(uniform(0,1));
dr2.l(t,nw) = round(uniform(0,1));

j1(nw) = (0.00708*k1*Fperf*h1)/(uo1*(0.5*log((Area1)/(rw1**2*Ca1))+5.75+s1));
j2(nw) = (0.00708*k2*Fperf*h2)/(uo2*(0.5*log((Area2)/(rw2**2*Ca2))+5.75+s2));

reservoirmodel.optfile=1;
option nlp=conopt3;
option iterlim = 10000;
option sysout = on;
Reservoirmodel.optcr=0.05;
Solve Reservoirmodel maximizing z using MIP;

zs.l(n) = z.l;
TT.l(n) = T12;
execute_unload "zs.gdx" zs.l
execute 'gdxxrw.exe zs.gdx var=zs.L'

Display pwf1.l,pwf2.l,p1.l,p2.l, z.l, Q1.l,Q2.l,QT.l, Np.l, dr1.l,dr2.l, Tcost.l, TotalOPEX.l, TotalCAPEX.l, Platformcost, Initialfacilitycap,
        Facilitycapacity.l, expansioncost.l,wellcost.l,maxrate1,maxrate2,OOIP1,OOIP2;
);
```

```
*option MINLP = GAMSCHK;
*Solve Reservoirmodel using MINLP maximizing z;

*$ontext
execute_unload "Np.gdx" Np.l
execute 'gdxxrw.exe Np.gdx var=Np.L'

execute_unload "P1.gdx" P1.l
execute 'gdxxrw.exe P1.gdx var=P1.L'

execute_unload "P2.gdx" P2.l
execute 'gdxxrw.exe P2.gdx var=P2.L'

execute_unload "dr1.gdx" dr1.l
execute 'gdxxrw.exe dr1.gdx var=dr1.L'

execute_unload "dr2.gdx" dr2.l
execute 'gdxxrw.exe dr2.gdx var=dr2.L'

execute_unload "Pwf1.gdx" Pwf1.l
execute 'gdxxrw.exe Pwf1.gdx var=Pwf1.L'

execute_unload "Pwf2.gdx" Pwf2.l
execute 'gdxxrw.exe Pwf2.gdx var=Pwf2.L'

execute_unload "Q1.gdx" Q1.l
execute 'gdxxrw.exe Q1.gdx var=Q1.L'

execute_unload "Q2.gdx" Q2.l
```

```
execute 'gdxxrw.exe Q2.gdx var=Q2.L'

execute_unload "QT.gdx" QT.l
execute 'gdxxrw.exe QT.gdx var=QT.L'

execute_unload "Qxf.gdx" Qxf.l
execute 'gdxxrw.exe Qxf.gdx var=Qxf.L'

execute_unload "z.gdx" z.l
execute 'gdxxrw.exe z.gdx var=z.L'

execute_unload "Tcost.gdx" Tcost.l
execute 'gdxxrw.exe Tcost.gdx var=Tcost.L'

execute_unload "TotalOPEX.gdx" TotalOPEX.l
execute 'gdxxrw.exe TotalOPEX.gdx var=TotalOPEX.L'

execute_unload "TotalCAPEX.gdx" TotalCAPEX.l
execute 'gdxxrw.exe TotalCAPEX.gdx var=TotalCAPEX.L'
*$offtext
```

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